MASARYKOVA UNIVERZITA
Přírodovědecká fakulta
Ústav teoretické fyziky a astrofyziky

Bakalářská práce

Brno 2021 Kateřina Neumannová

MASARYKOVA
UNIVERZITA
PŘÍRODOVĚDECKÁ FAKULTA
ÚSTAV TEORETICKÉ FYZIKY A ASTROFYZIKY

Hvězdná proměnnost v kulových hvězdokupách

Bakalářská práce

Kateřina Neumannová

Vedoucí práce: Mgr. Marek Skarka, Ph.D. **Brno 2021**

Bibliografický záznam

Autor:	Kateřina Neumannová Přírodovědecká fakulta, Masarykova univerzita Ústav teoretické fyziky a astrofyziky
Název práce:	Hvězdná proměnnost v kulových hvězdokupách
Studijní program:	Fyzika
Studijní obor:	Astrofyzika
Vedoucí práce:	Mgr. Marek Skarka, Ph.D.
Akademický rok:	2020/2021
Počet stran:	viii + 40
Klíčová slova:	Kulová hvězdokupa; Proměnná hvězda; Metalicita

Bibliographic Entry

Author:	Kateřina Neumannová Faculty of Science, Masaryk University Department of Theoretical Physics and Astrophysics
Title of Thesis:	Stellar variability in globular clusters
Degree Programme:	Physics
Field of Study:	Astrophysics
Supervisor:	Mgr. Marek Skarka, Ph.D.
Academic Year:	2020/2021
Number of Pages:	viii + 40
Keywords:	Globular cluster; Variable star; Metallicity

Abstrakt

Tato bakalářská práce se věnuje základním vlastnostem kulových hvězdokup a proměnných hvězd, které se v nich nacházejí. Z dat jsme zjistili, že kulové hvězdokupy mají bimodální rozdělení s ohledem na metalicitu. Středy dvou populací odpovídají hodnotám metalicit $-1,48$ a $-0,49$. Závislost věku na metalicitě nám ukázala lineární závislost pro hvězdokupy v galaktickém halo, a také v galaktické výduti. Metalicita hala a výdutě se liší o 0,86 dex, při stejném stáří, s tím, že kulové hvězdokupy v halo mají nižší metalicitu. U proměnných hvězd jsme se zaměřili na jejich typy, počty hvězd jednotlivých typů ve hvězdokupách, na metalicitu a věk kulových hvězdokup jež obsahují dané typy. Proměnné hvězdy v kulových hvězdokupách se nachází u všech hodnot metalicit. Nejvíce RR Lyrae se nachází poblíž metalicity $-1,4$. U SX Phe je náznak, že jejich množství stoupá k nižším metalicitám, zatímco u pomalých nepravidelných proměnných hvězd (L typ) tomu je naopak. U hvězd typu RR Lyrae a SX Phoenicis se ukázaly lineární závislostí průměrné periody na metalicitě, a také věku na metalicitě. U ostatních typů nebyly závislosti vždy patrné.

Abstract

In this thesis, we study the basic properties of globular clusters and variable stars inside them. We noticed, that the metallicity of clusters have bimodal distribution. The centers of the two populations correspond to metallicity value of -1.48 a -0.49. The age-metallicity relation shows linear dependencies for the globular clusters located in galactic halo and for the galactic bulge. The globular clusters located in halo are more metal deficient of 0.86 dex than in the bulge at the same age. For variable stars, we focused on the occurence rate of the variable types, the metallicity and age of the globular clusters. It was found out that the variable stars in globular clusters are located in a whole range of metallicities. The most variable stars of RR Lyrae type are located close to metallicity -1.4. There is an indication for SX Phe (SXPHE) variables that their number increases to metal-deficient clusters, while for slow irregular variable stars (L type) it is the other way around. RR Lyrae and SX Phoenicis variables show linear dependencies for period-metallicity relation and age-metallicity relation. The dependencies were not always apparent for the other types of variable stars.

MASARYKOVA UNIVERZITA
PŘÍRODOVĚDECKÁ FAKULTA
KOTLÁŘSKÁ 2, 611 37 BRNO
IČ: 00216224
DIČ: CZ00216224

ZADÁNÍ BAKALÁŘSKÉ PRÁCE

Akademický rok: 2020/2021

Ústav:	Přírodovědecká fakulta
Studentka:	Kateřina Neumannová
Program:	Fyzika
Obor:	Astrofyzika

Ředitel *ústavu* PřF MU Vám ve smyslu Studijního a zkušebního řádu MU určuje bakalářskou práci s názvem:

Název práce:	Hvězdná proměnnost v kulových hvězdokupách
Název práce anglicky:	Stellar variability in globular clusters
Jazyk závěrečné práce:	angličtina

Oficiální zadání:
Proměnné hvězdy tvoří základní zdroj informací o procesech, které se odehrávají ve hvězdách, na jejich povrchu, a v jejich okolí. Cílem práce je získat přehled o výskytu různých typů proměnnosti v kulových hvězdokupách a odhalit tak jeho případnou závislost na metalicitě a věku.

Vedoucí práce:	Mgr. Marek Skarka, Ph.D.
Datum zadání práce:	30. 11. 2020
V Brně dne:	7. 6. 2021

Zadání bylo schváleno prostřednictvím IS MU.

Kateřina Neumannová, 21. 1. 2021
Mgr. Marek Skarka, Ph.D., 26. 1. 2021
Mgr. Michael Krbek, Ph.D., 1. 2. 2021

Poděkování

Na tomto místě bych chtěla poděkovat mému vedoucímu Mgr. Marek Skarka,PhD. za cenné rady a vedení mé práce, také za jeho vstřícný přístup po dobu psaní mé práce. Zároveň bych chtěla poděkovat všem, kteří mě v průběhu mého studia a psaní práce podpořili, a to především mé rodině. V neposlední řadě bych ráda poděkovala Ing. Miroslav Šmíd, Ph.D. a Mgr. Iva Holasová za věcné rady a případné poznámky k mé práci.

Prohlášení

Prohlašuji, že jsem svoji bakalářskou práci vypracovala samostatně pod vedením vedoucího práce s využitím informačních zdrojů, které jsou v práci citovány.

Brno 16. června 2021

. . . . . .

Kateřina Neumannová

Contents

Introduction

The first observed variable star was captured during the explosion of supernovae in the constellation of Taurus. It was thought that this explosion was an atmospheric phenomenon but Tycho Brahe (the 16th century) had a different opinion. Tycho Brahe recorded the light curve of supernovae in the constellation of Cassiopeia and he thought that it was astronomical phenomena occuring far in the sphere of stars. It was the first step to the search for variable stars and it was found that there are more types of variable stars (e.g., pulsating, rotating, etc). Variable stars are a rich source of information, such as information about radius, inner composition, mass, luminosity, and distances in the universe (Cepheids). Some types of variable stars are also located in globular clusters.

The term "globular cluster" first appeared in the 18th century. Charles Messier and Herschel were the first to mark an object as a globular cluster and thus, the research of globular clusters began. Currently, 157 globular clusters are known in the Galaxy. Globular clusters are often studied in astronomy because the stars inside them are of the same age (they are very old), have the same chemical composition and are gravitationally bound. Consequently, we can estimate the minimum age of the universe (from the oldest clusters), the evolution of stars, and what the universe looked like at the time of its formation.

This thesis aims to compare and show correlations and dependences of variable stars inside globular clusters but it does not replace detailed articles focused exclusively on a few types of variable stars or several globular clusters. Rather, it opens the possibility for further and deeper analysis of both globular clusters and variable stars.

The chapter Variable stars deals with variable stars in terms of methods of their research or their classification and types. The second chapter focuses on globular cluster (Globular clusters) and the variable stars inside them. The chapter Methodology and data processing deals with sources of data and data processing, and the process of database creation. Analysis was the next part of the thesis, where the correlations were investigated. The last part sums up the thesis in Discussion and summary part.

Variable stars

1.2 Definition

Variable stars are stars that change their brightness in time. Their brightness variations range from micromagnitude to more than 10 magnitudes (Harrison & Campbell, 2018). We can also observe a wide range of time variability, whose values are in the range from 10^{-4} seconds (Hessels et al., 2006) to millions or billions of years (the nuclear evolution of stars).

Each star can be variable and depends only on the accuracy of the measurements, technique, and the time scale, which we measured it with. As technology improves, we can measure more accurately and discover more variable stars. The slowest changes in variability are caused by nuclear evolution at the center of star and take about 10^6–10^9 years. Astronomers who are interested in variable stars usually work with star variability, which changes faster. It is not usual to deal with changes in the order of millions or billions of years.

1.3 Brief history of the search for variable stars

One of the first observed variable stars was κ Cas by Tycho Brahe (1572). He constructed the first light curve of a variable star ever. David Fabricius observed Omicron Ceti in 1596. Fabricius noticed that the brightness of the observed star is changing and named it Mira (strange). His discovery was followed by Ismael Boulliau who found out the periodicity of Mira. He measured a period of 333 days. Now we know that he was very close to our accepted value which corresponds to 332 days. In the 18th century, Edward Pigott and John Goodricke assembled the first catalogue of variable stars that contained 12 items. These included Nova Cas, Algol, Mira, β Lyrae, δ Cephei, χ Cygni, R Leo, Nova Vul, P Cygni, Nova Oph, R Hya and η Aquilae. In 19th century, Friedrich Argelander devised a way to estimate the brightness of variable star. He used relative comparison of a variable star with star featuring comparable brightness, which were located in the immediate vicinity. Some of the amateur astronomers still use this technique.

In the 19th and the 20th century, there was a breakthrough in astronomy. Photographic plates and later CCD cameras (charged coupled device) were brought to the fore because the human eye had become insufficient for observation and research. At the end of the 20th century, satellite observations, came forward. Thanks to satellite Hiparcos, 8000 (e3) , new variable stars were discovered. In the 21th century, mainly measurements from satellites *GAIA* (e4), *MOST* (e5), Kepler (e6) are used for the photometric research of

variable stars. Variable stars are also examined by ground-based observatories, which include ASAS, ROTSE, OGLE, and others (e30). The known variable stars including the basic information of the stars are listed in the VSX database website aavso.org, where the information is up-to-date (Watson et al., 2007).

1.4 Methods for variable stars research

We can ask ourselves, why we are interested in variable stars actually and why it is useful to study them. First, something is going on inside them and/or around them. They are not static objects. We can observe changes of brightness which have a wide range depending on type of variable star.

For the research of variable stars, the light curve is very important. Light curve is the dependence of the brightness on time. During one observation, we are not always able to measure the whole brightness change cycle, therefore, we have to merge more parts of light curves into phase-folded light curve. From these curves it is possible to estimate physical properties of the stars such as the type of variable star, parameters of variability, confirm or refute the period we have specified and get suspicion of another object. Thanks to novae, supernovae, eclipsing binaries, pulsating variable stars, and others, it is possible to determine distances in space.

The brightness of a variable star is usually measured by photometer, which is a device able to detect light (photons). Photometers may contain several photometric system filters. Using photometric filters, we can detect distribution of energy in spectrum that depends on the surface temperature of the star. The most extended photometric system is Johnsons photometric system (Johnson & Morgan, 1953), with filters defining the brightness in colors: U (365 nm), V (550 nm), B (440 nm). Johnson photometric system was extended by Cousins (Cousins, 1976). Cousin added filteres in color R (634.9 nm) and I (879.7 nm). Many system exist such as Stromgren's (Strömgren, 1956) and Sloan's (Fukugita et al., 1996).

Another method for variable star research is the spectroscopic method. It is one of the oldest, most valuable and most important methods for examining an object. This method can be used to determine whether a star is single or binary, what kind of star it is (dwarf, giant, etc), whether it contains a strong magnetic field, etc. Three main types of spectra are: continuous, absorption and emission, see Fig. 1.4.1. The continuous spectrum is formed by all wavelengths from a certain range and arise in hot and dense substances. The absorption spectrum arises when the continuous passing light is absorbed by colder gasseous substance (atmosphere of stars, nebulae). The substance absorbs the light, due to electrons move to higher energy levels. Electrons do not stay at this level for long time. They are stransfered to a lower level and emit a photon of appropriate energy. This is called emission and corresponding spectrum is called emission spectrum. The stellar spectra are created in stellar photospheres and they are connected with absorption, emission and continuous spectra. Absorption lines are formed when light arrives through the colder photosphere and emission spectra dominates in chromosphere (e9) (e10).

The last observation technique I want to mention is polarimetry. This field is built on the fact that the light is polarized. Light is electromagnetic wave described by electric intensity $\vec{E}$ and magnetic induction $\vec{B}$. These vectors are perpendicular to the direction in which the

waves are spread, while at the same time $\vec{B}$ is perpendicular to the $\vec{E}$ and vice versa. If we limit the oscillation direction of vector $\vec{E}$ or $\vec{B}$, we get polarized light. Polarized light carries information about the nature of glowing object and arise in many cases by dispersion (from the Sun or the Moon), reflection or refraction (interstellar medium). Polarimetric measurements are useful for tight binary stars (determining inclinations towards Earth) or chemically peculiar stars (Zeeman effect – splitting spectral line because of static magnetic field). Polarimetric measurements are time-demanding. The reason is, that we need a long exposure time (polarization is weak, thus it is necessary to have an efficient apparatus with high transmittance) and the background can be contaminated by polluted light, zodiacal light etc.

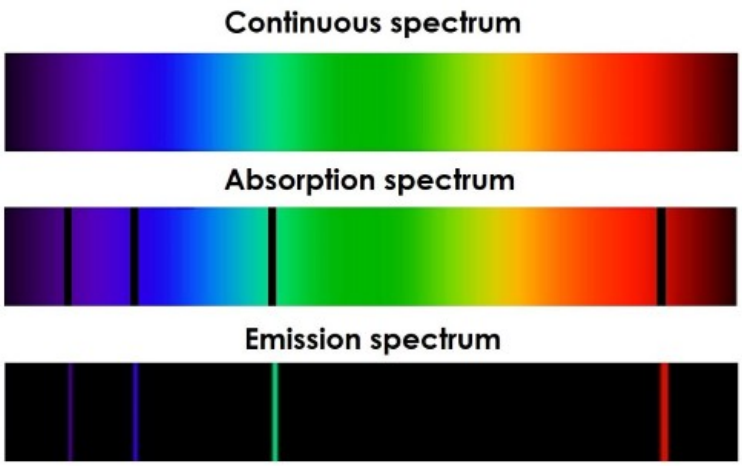

Figure 1.4.1: Continuous, absorption and emission spectra (e20).

1.5 Basic classification

Variable stars may be either intrinsic or extrinsic. Intrinsic variable stars are called real/physical variable stars. Their variability of physical properties is caused by the stars themselves. Luminosity of a star varies. There are three basic subgroups of intrinsic variable stars: eruptive, pulsating and cataclysmic or explosive variables. Extrinsic variable stars are stars whose luminosity does not change, only their brightness does. There are two subgroups of extrinsic variable stars: rotating and eclipsing binaries. More detail classification you can see in Fig. 1.5.1. In the next chapter we will describe the individual types of variable stars in more detail.

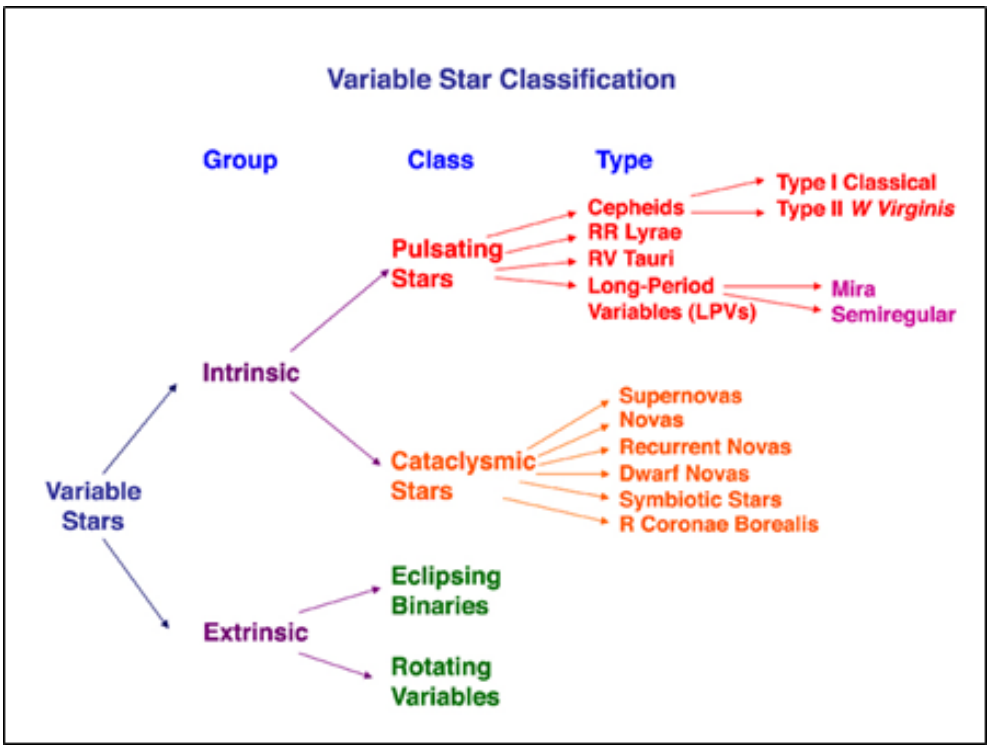

Figure 1.5.1: Variable star classification (e21).

1.5.1 Extrinsic variable stars

Rotating variable stars

As the stars rotate, we observe brightness variations. The radiation must deviate from strictly axial symmetry. Rotating variable stars change their brightness because of surface spots or/and fast rotation. Reason of fast rotation is, that the star is at the beginning of its evolution (it is shrinking, trying to maintain its momentum). Another explanation is, that the star is part of binary system. As a result of fast rotation, the star has a shape of an ellipsoid (star is deformed) and that causes brightness variation. The size of the surface, which is oriented towards the Earth, changes. Spots can be caused by strong magnetic field. Stars of solar type and cooler stars contain cold spots in stellar parts with strong local magnetic field. Chemical peculiar stars contain spots in areas with different representation of elements. Spots have the same temperature as the surroundings but different spectral composition. We have basic types of rotating variable stars. These are Solar-type stars (main-sequence stars), FK Comae Berenices (cold giant stars), RS Canum Venaticorum (binary stars), BY Draconis (main-sequence stars), Pulsars (collapsed stellar cores of supergiants) and Chemically peculiar stars (main sequence stars).

Eclipsing binaries

Binary system represents two objects, orbiting the common center of mass. We observe decreases in brightness on the light curve of binary system if the system is oriented

properly. These decreases are caused by transition or occultation. Transition occurs when the smaller star(object) passes in front of a larger star(object). The occultation is the opposite phenomen.

If components of binary system are close enough, we can talk about close binary stars. All points on an equipotential surface have the same potential. The result is that the work done between two points on an equipotential surface is zero (e11). These surfaces we call the Hill sphere for each component of close binary stars. If the object is homogeneous these spheres are ball-shaped, otherwise the sphere is deformed. Hill sphere, which is common for both stars is called a Roche lobe. A Roche lobe forms around binaries, which determines the behavior of matter and objects inside. An object located in the Roche lobe of a star is subject to and orbiting its gravitational field. The shape of Roche lobe depends only on the ratio of masses. In the case where the star fills the Roche lobe, the material begins to overflow to the second component through lagrangian point 1 (L_1), Fig. 1.5.2. If both lobes are filled, the material also escapes through $L_2$and L_3. We have Kopal´s classification (Kopal, 1955) which divides binary stars according to the relation to the Roche lobe. These are: detached, semi-detached and contact system. We have several types of eclipsing binary variable stars divided by the shape of the light curve. These are EA Algol (detached), EB beta Lyrae (semi-detached) and EW Ursae Majoris (contact) 1.5.3.

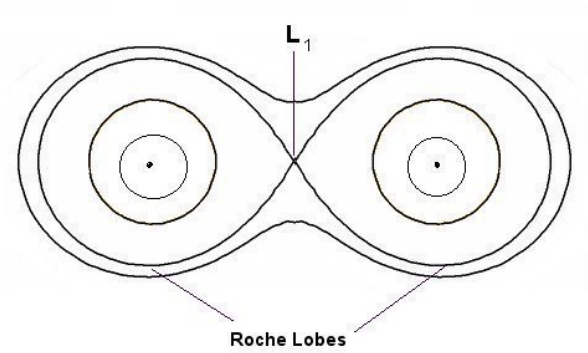

Figure 1.5.2: Roche Lobes and Langragian point L_1 (Illingworth, Valerie. The Facts on File Dictionary of Astronomy, Third Edition, 1994, pg. 146)

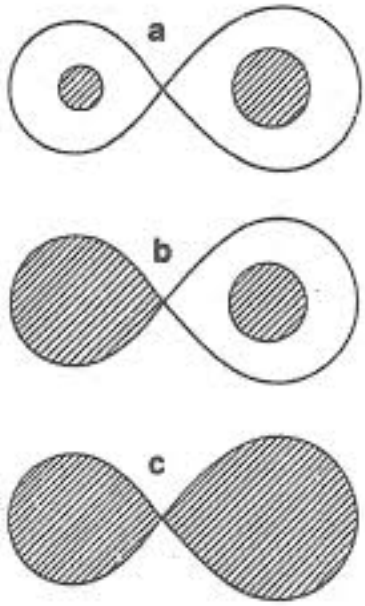

Figure 1.5.3: Kopal's classification, a = detached system, b = semi-detached system, c = contact system (e22).

1.5.2 Intrinsic variable stars

Cataclysmic or explosive variables

These types of intrinsic variable stars can be called tight interacting binaries. Sudden brightening is primarily the result of explosions, e.g., supernovae. Another reason of uneven radiation is caused by deformation of components or by the reflection effect. The reflection effect is based on the mutual illumination of the stars. It causes partial absorption of radiation which leads to the heating of the outer layer and partial scattering radiation into space. Brightness facing each other part of the stars is greater than the opposite parts of the surfaces. Deformation of components occurs during the rotation of the stars around a common center of mass.

Cataclysmic variables contain a star from the main sequence and a white dwarf. Their orbital periods are within hours. The star from the main sequence loses mass and passes it by accretion to the white dwarf. Hydrogen predominates during accretion, star warms up to the temperature at which the thermonuclear reaction is triggered. Then the nova explosion occurs. Symbiotic variables contain red giant and white dwarf. The red giant transfers matter to the white dwarf by a stellar wind. The white dwarf ionizes stellar wind and a symbiotic nebula is formed around the system. The change in brightness has periods about hundreds of days.

Pulsating variables

They make up the majority of all variable stars (e1). Pulsations can be radial and non-radial. Radial pulsations occur where the entire star expands and shrinks (the radius changes).

Non-radial pulsations are pulsations where one part of the star expands while another part shrinks (the shape changes). Due to pulsations, the effective temperature of the stars changes. During pulsations their hydrostatic balance is disturbed. In the contracted state, a pressure gradient predominates, otherwise gravity prevails. The pulsations correspond to longitudinal wave (the displacement of the particles is in the same/opposite direction as, the direction of propagation of the wave) which interferes with themselves and create standing waves. There are nodes in the center of star and antinodes on its surface. Stars pulsate in modes that have different frequencies. These modes are fundamental-frequency mode which determines the period of the fundamental mode and a higher-frequency mode (harmonic/overtone).

Schwarzschild (1940) found out a connection between modes of pulsation and the shape of light curve. It means that for the fundamental mode we have a higher amplitude and period. The light curve is asymmetric. For the first harmonic mode there is lower amplitude and period, but the light curve is sinusoidal there. We also know the pulsation equation, which tells us that the period of the star's fundamental oscillation is directly proportional to the inverse of the square root of the mean density of the star e.q. 1.1.

$$P = \sqrt{\frac{1}{\rho}} \tag{1.1}$$

Pulsations are damped by viscosity and friction.

Pulsations are oscillations around the equilibrium position. Normally, a star is in the hydrostatic balance (i.e., gravitation and pressure gradients are in balance). Each star has a layer of once and twice ionized helium (He I and He II). This layer must be placed at the correct depth for the radial pulsations to begin, otherwise the pulsations will be damped. If the layer is close enough to the surface, the first harmonic modes appear. In contrast, if the layer is deeper than the one in which the first harmonic mode arises, the fundamental modes appear there. Otherwise, it is not the only condition. The radial pulsation mode is described by parameter n. The fundamental mode corresponds to $n = 0$, for the first harmonic we have $n = 1$. Radiation and convection (movement of matter) are the sources of pulsation.

If the layer is not composed of He I and He II, then opacity (κ) decreases with compression as temperature (T) increases (Kramer opacity, e.q. 1.2). The ρ means density.

$$\kappa = \rho T^{-3.5} \tag{1.2}$$

Layers of ionized helium work differently. Radiant flux through the layer causes some of the radiation to be spent on ionizing helium atoms rather than heating. Density and opacity rise. The layer does not transmit so much radiation and it accumulates under the layer. Pressure rise, the layer absorbs more energy, and the layer expands. Density and opacity decreases. Helium atoms recombine. Considering the surrounding layers, the layer transmits more radiation and lacks energy for further expansion. This leads to shrinkage of the layer and, therefore, of the whole star.

In non-radial pulsation, the wave does not penetrate into the center of the star, but only into the inner parts of star. Towards the inner parts of the star, the temperature of the star material increases, wave speed increases and a refractive index decreases,

therefore, the wave can refract back to the surface. If the wave get there, it can reflect on the photosphere, see Fig. 1.5.4. Wave interference creates standing waves, which cause non-radial pulsations.

Stars pulsate both with sound waves in pressure modes and/or in gravity modes. Gravity modes are in the inner parts of a star and they have long periods. Restoring force is gravity in the case of gravity modes. Pressure modes occur in the whole star and have a short periods (the Sun has a 5 minute oscillations). In the case of pressure modes, the restoring force is pressure. Non-radial pulsations are described by parameters n, l, m. Parameter l is equivalent to the number of node circles on the surface of the star. Parameter m is an azimutal number, which represents waves travelling around the star. Parameter n characterizes the number of nodes in the radial part, from the center to the surface. These quantum numbers concerns the inside of the star in the case of parameter n, and the surface of the star in the case of parameters l, m.

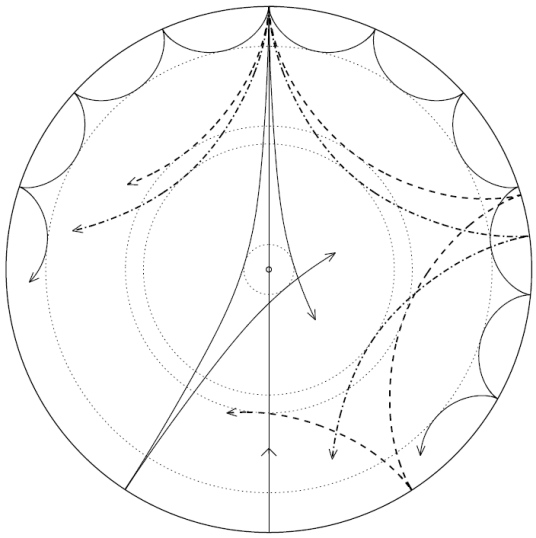

Figure 1.5.4: Mode penetration through the stellar interior. Waves behave according to the Snell's law. The angle at which the wave reflects determines how far it will penetrates: The lower the angle, the shallower the penetration. The density, temperature and the speed of sound increases, thus the refractive index decreases. It causes that the wave goes back to the surface (e23).

Pulsating stars include Cepheids, W Virginis, RR Lyrae, Mirids, etc. Some of the types that can be found in globular clusters are described in section 2.1.

Globular clusters

2.1 Definition

Globular clusters from the Latin word globulus (a small ball) are tightly packed, spherical collection of stars as you can see in Fig. 2.1.1. Due to being tightly bound by gravity, density of stars increases towards their centers. Most stars in globular clusters are at the same stage of stellar evolution. However, some stars show different ages as a consequence of meeting a molecular cloud, which is necessary for star formation or a fusion of more clusters. As a result, the next generation of stars was formed. This has led to some globular clusters have a bimodal or multimodal population (bimodal color distribution) of clusters.

Figure 2.1.1: Globular Cluster of Omega Centauri (e24).

Globular clusters were formed at the same time as their maternal galaxies, so they are one of the oldest objects in the universe, dating back billions of years. Their age can be established from an Hertzsprung-Russel diagram (HR diagram), which represents the relation between the absolute magnitude of stars on the vertical axes and the color index on the horizontal axes. The temperature of a star determines the color index. The smaller the color index, the hotter and bluer the star is. The mostly used color index is (B-V) connecting the colors in blue and visual Johnson filters. In short, a color index measures the difference of two brightnesses in an interval of wavelengths.

On the HR diagram of stars from a globular cluster, we can observe a main sequence without the stars of spectral type OBA and a huge parts of giants and white dwarfs. Due to a short main sequence, we estimate, that stars in globular clusters are very old. In Fig. 2.1.2, we can see a short main sequence with a turn-off point at around (BV) = 0.3 which corresponds to a temperature of approx. 6000 K (spectral type G). That means that spectral type OBA are already gone from the main sequence and we can estimate the age

of these stars based on the knowledge of the time of star evolution of a different spectral type.

Old stars contained in globular clusters are from population II. These stars are metal-poor, because the iron-peak elements were very rare in the time globular clusters were formed. Inside clusters, molecular dust and gas have not been observed, so we can expect that new star will not arise there. Clusters are located in the Galactic halo and the Galactic bulge. The reason is that Galactic halo and bulge were formed by the collapse of primordial clouds at the beginning of galaxy formation, thus the first stars appear there. The halo contains less metals compared to the Galactic bulge. This may mean that Galactic halo is younger than Galactic bulge. Between stars in a globular cluster, there is a gravitational interaction. It means, they are gravitationally bound. Thanks to that, the globular clusters have been stable for billions of years.

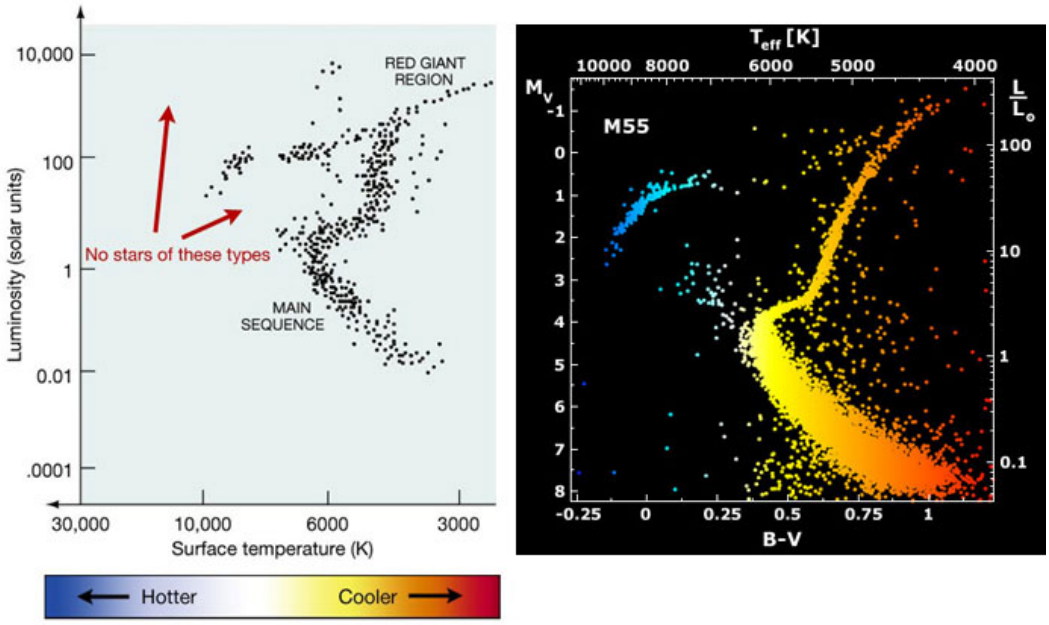

Figure 2.1.2: Globular cluster - HR diagram. On the left diagram we can notice, there are no stars of OBA type. On the right-hand panel, there is as an example HR digram of M55 with distinct areas of red giant branch, extreme horizontal branch and main sequence (e25).

2.1.1 Comparison with open clusters

Globular clusters were formed in a halo around the galactic core with distances from the Sun larger than 2000 pc (e2). Most of them can be found in the Sagittarius, Scorpio and Ophiunchus constellations (in the Galaxy). The mass of a globular cluster can be up to $1x10^6$ $M_\odot$ (D'Souza & Rix, 2013) and globular clusters are very old, their age corresponds a billions of years. On the other hand, open clusters were formed in the spiral arms. Their masses are lower: below 50 000 M Sun (e2). Open clusters are younger than globular cluster, their age is up to 5 billion years, however, the age of some open clusters can reach up to 10 billion years (Salaris et al., 2004). Their young age guarantees a higher metal abundance compared to globular clusters.

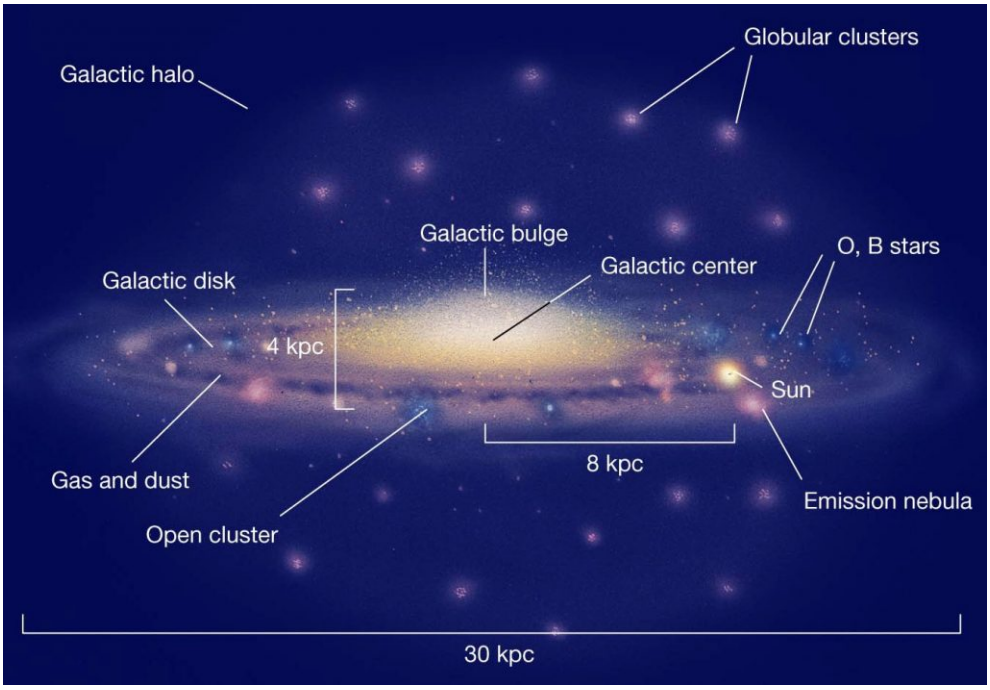

Figure 2.1.3: Galaxy and location of globular and open clusters (e26).

2.1.2 Brief history of globular cluster observation

The first globular cluster was noticed by Ptolemy (150 AD) who, however, was not aware that it was a cluster. He named it "a star on the horseback". James Dunlop (1826) rediscovered this object and described it as a cluster. Now this cluster is known as Omega Centauri. Globular cluster M22 was discovered by Abraham Ihle (1665), who also labeled this object incorrectly, as a nebula. William Herschel and Charles Messier, were one of the first to designate these objects as globular clusters. They also observed and identified new globular clusters and catalogued them. To date (31.1.2021), 157 Galactic globular clusters are known (e7).

2.2 Variable stars in globular clusters

Variable stars in globular clusters can help us to determine the distances to the globular clusters because they can be used as standard candles (Cepheids, RR Lyrae). They are also necessary for the understanding of the age, structure, and formation of the Galaxy. The globular clusters are very old, thus the variable stars inside them are at an advanced stage of their lives or they are the small-mass stars of the main sequence.

Since 2001, 3000 variable stars were known inside the Galactic globular clusters (Clement et al., 2001). Most of variable stars are of the RR Lyrae type with 1842 members. Slow variables (L, SR, M) contain 117 members, the same goes for the SX Phoenicis. Eclipsing binaries have 90 members and W Virginis, RV Tauri just have 60 members. The values were taken for the year 2001 from (Clement, 2017) . This article also conducted a research on the properties (such as period) of variable stars in globular clusters. Most variable star contain Omega Centauri cluster shown in Fig. 2.1.1.

2.2.1 RR Lyrae

Willemina Fleming noticed a change in the brightness of a star in the Lyra constellation. This star was named RR Lyr. Some stars in globular clusters had a similar light curve and period, hence they were denoted as variable stars of the RR Lyrae type. RR Lyrae stars are radially pulsating variable stars, which can also pulsate non-radially. They belong to the group of horizontal-branch giants and lie in the instability strip corresponding to spectral type A-F. These stars can be divided into three groups according to the mode in which they pulsate.

The majority of variable stars consist of RR Lyrae stars. Data estimate RR Lyrae account for approximately 17% (Dolinsky, J., 2017) (Variable Star Index Catalogue, Watson 2006), of all types of variable stars.

The three basic groups are RRa, RRb, RRc. Theoretical calculations proved that the RRa and RRb pulsate in the fundamental mode, so in the modern catalogues we can find only one RRab group (Bailey, 1902). RRc pulsate in the first overtone mode. There are also RRd and RRe groups. RRd pulsate in the fundamental and the first overtone modes, while RRe pulsate in the second overtone mode. The light curves of RRab and RRc are quite different, see Fig. 2.2.1. The RRab type has an asymmetric light curve which can be identified by rapid ascension and slow descension from the maximum light. The RRc has a symmetric light curve with a sinus-like shape.

The light curve of many of RR Lyrae stars is modulated in such a way that its amplitude or period changes periodically, or there are changes in both parameters at the same time. It takes several pulsation cycles for the light curve to achieve the same shape. This effect is called the Blazhko effect (Blažko, 1907) and so far, there is no satisfactory explanation for it (Kolláth, 2018).

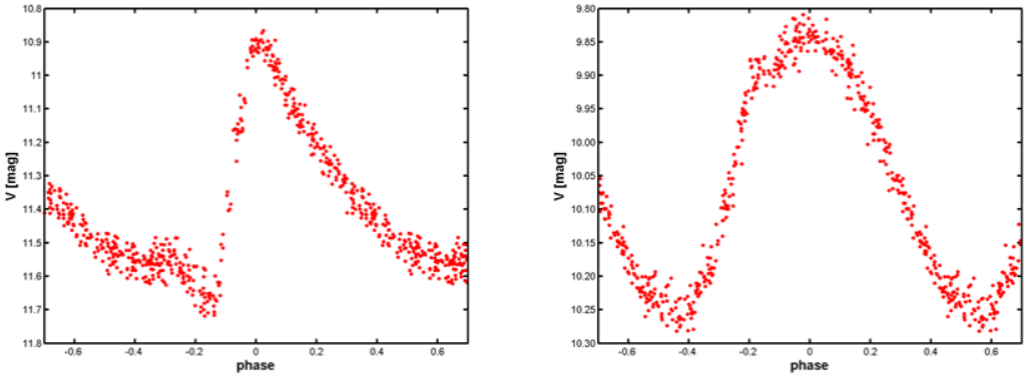

Figure 2.2.1: Typical light curves of RR Lyrae type stars.The left panel shows the RRab type, while the right-hand panel shows the RRc type. (e1), page 186

2.2.2 Long-periodic variable stars

They are intrinsically variable cool giants or supergiants. Long periodic variable stars contain Mira variables, Semiregular variables, type RV Tauri and irregular variables (Sipahi, 2012). Semiregular variables can be divided into SRa, SRb, SRc and SRb type. All these types have diverse shape of light curves but all of them show a noticeable periodicity sometimes interrupted by some irregularity. RV Tauri are radially pulsating super/giants. The light curve has 2 minima, and the depths of minima may vary. Irregular variables are slow variable stars of late spectral type whose light changes do not show any periodicity. Irregular variables are Lb and Lc type. Lb type are giants whereas Lc type are supergiants.

Mirids

Omicron Ceti from the Cetus constellation was the first systematically observed variable star. These are long-period pulsating cold stars from the asymptotic giant branch with large luminosity. There are two main sources of pulsations. These are processes in the hydrogen/helium layers and 3-alpha process (Kilston, 1975). The 3-alpha process fuses three helium nuclei to form carbon and oxygen nuclei. The reactions are very fast and huge, which causes a shock wave as a result of reaching the flash point of nuclear reactions. The shock wave rises to the surface and heats up all the layers which causes a dissociation of molecules, decreases the opacity, and increases the brightness of the star. After some time, the opacity increases again and the brightness decreases. The pulsations are quite periodic. To illustrate periodic changes in brightness I use light curve of R Cas Fig. 2.2.2.

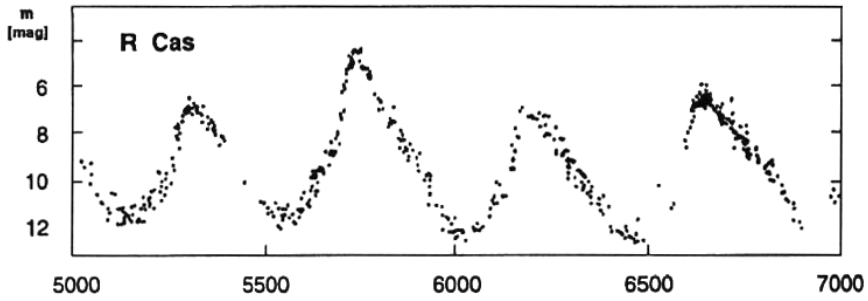

Figure 2.2.2: Light curve of Mirids and the variable star R Cas (e27).

2.2.3 W Virginis

These stars are old giants with less distinct luminosity-period relation than Cephei stars. This means that we can use them for measuring the distances, although with less precision than using classical Cepheids. Their pulsation periods are 10-20 days (Wallerstein, 2002) with an amplitude of up to 2 mag (e8). W Virginis have low masses and metallicities and lie in the instability strip of the HR diagram. The light curve is quite periodic, however, there are humbs between two minima Fig.. 2.2.3 .

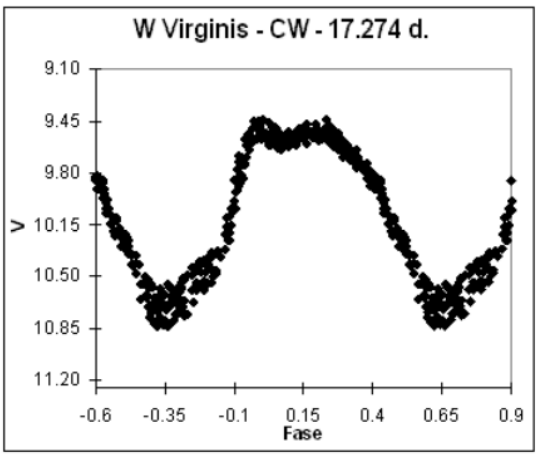

Figure 2.2.3: Light curve of W Virginis with period 17.3 days (e1), page 185.

2.2.4 SX Phoenicis

SX Phe are short-period variable subdwarfs with low metallicities. Period and metallicity are correlated in SX Phe stars. Stars with shortest period are found in the metal-poor clusters (McNamara, 1995). SX Phoenicis stars show period-luminosity relations and all of them are located in the blue straggler region (Jeon et al., 2004). They are more luminous and hotter (bluer) than the other stars from cluster. The possible explanations are, that the stars were formed later than the others, the stars are not actually part of the cluster or it is the effect of collisions of stars (Leonard, 1989).

2.2.5 Eclipsing binaries

There are 3 main group of eclipsing binaries. These are EA – Algol type, EB – Betta Lyrae type and the last one EW – W Ursae Majoris type. Decreases in brightness are caused by eclipses of components.

EA = Algol type is semidetached system where the components are a main-sequence star and a cooler, larger subgiant. Except the occultation, the brightness of the system does not change or very slightly as you can see in the top panel of Fig. 2.2.4.

EB = Stars of β Lyr type are contact systems, which means, that there is mutual overflow of the matter between the components. Components are deformed and a reflection effect may be observed (Vaz, 1985). Reflection effect occurs as a result of close systems in which the components irradiate each other. Primary and secondary minima on the light curve are similar and quite smooth Fig. 2.2.4. It is not easy to find exact moment when an eclipse begins and ends.

EW = This type of contact system has primary and secondary minima equally deep. We are not always able to determine beginning and the end of the eclipse as you can see in Fig. 2.2.4. W Ursae Majoris have common atmosphere and the components are in touch.

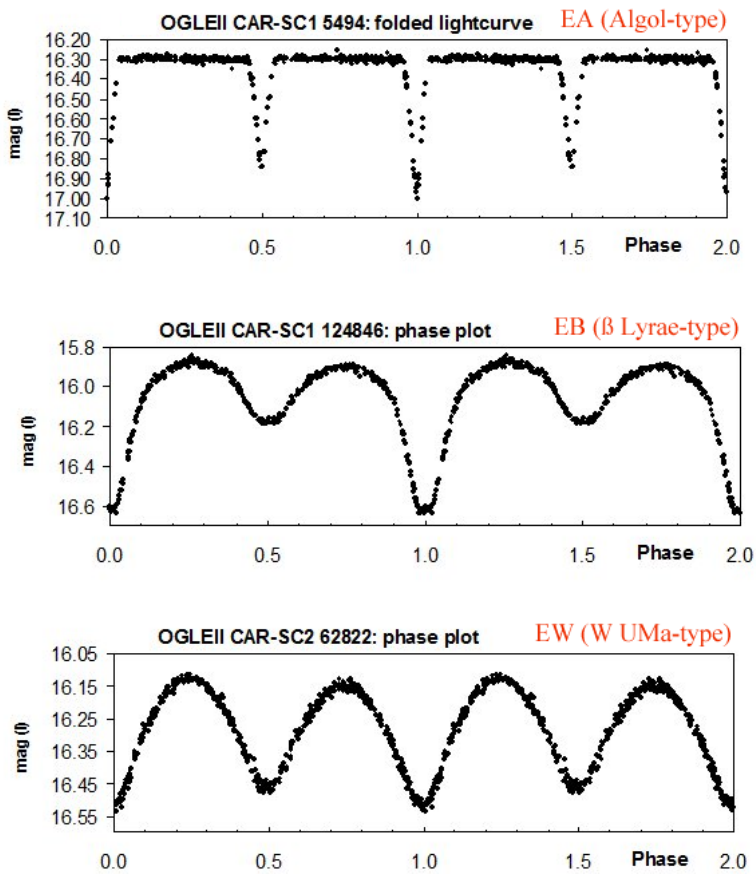

Figure 2.2.4: Light curve of eclipsing binaries. There we can see different light curves for each type of eclipse binary stars (e28).

Methodology and data processing

3.1 Data sources

The main goal of this thesis was to reveal a possible connection between different types of variable stars in globular clusters and metallicity, age of globular clusters and other parameters whose belonging to variables such as period. I used data available from the databases (e14) and (e29). The Fig. 3.1.1 shows the database from (e29). These databases contain data on 149 Milky Way Galaxy globular clusters and over 8000 variable stars. Some globular clusters in the databases include more than 200 variable stars (g.e., Omega Cen); on the other hand, some globular clusters do not contain information about variable stars at all (g.e., the data were missing for Palomar 1, Palomar 2, AM 1, Eridanus and some others). These clusters may contain variables but have not been detected yet. Each globular cluster has a list of variable stars containing their basic properties such as periods, types of variability, magnitude, see Fig. 3.1.2. The website (e14) includes important data on globular clusters e.g., metallicity, core radius, or half-light radius.

N104 (47 Tuc)	N288	N362	N1261	Pal 1
N1851	N1904 (M79)	N2298	N2419	Pyxis
N3201	Pal 4	N4147	N4372	Rup 106
N5053	Omega Cen	N5272 (M3)	N5286	AM 4
IC 4499	N5824	Pal 5	N5897	N5904 (M5)
Lynga7	Pal 14	N6093 (M80)	N6121 (M4)	N6101
N6171 (M107)	ESO 452	N6205 (M13)	N6218 (M12)	N6229
Pal 15	N6266 (M62)	N6273 (M19)	N6284	N6287
N6325	N6341 (M92)	N6333 (M9)	N6342	N6356
Ter 2	N6366	N6362	HP 1	Ter 4
N6388	Ton 2	N6402 (M14)	N6401	N6397
Ter 5	N6440	N6441	Ter 6	N6453
Djorg2	N6517	Ter 10	N6522	N6535
N6544	N6541	2MS-GC01	ESO-SC06	N6553
Ter 12	N6569	N6584	N6624	N6626 (M28)
N6652	N6656 (M22)	Pal 8	N6681 (M70)	N6712
N6749	N6752	N6760	Ter 7	N6779 (M56)
Ter 8	Pal 11	N6838 (M71)	N6864 (M75)	N6934
N7089 (M2)	N7099	Pal 12	Pal 13	N7492
AM 1	Eridanus	Pal 2	Liller 1	N6380
N2808	E3	Pal 3	Pal 6	N6426
N4590 (M68)	N4833	N5024 (M53)	UKS 1	N6496
N5466	N5634	N5694	N6528	N6539
N5927	N5946	N5986	2MS-GC02	N6558
N6144	N6139	Ter 3	N6638	N6642
N6235	N6254 (M10)	N6256	N6715 (M54)	N6717 (Pal 9)
N6293	N6304	N6316	Pal 10	Arp 2
N6355	N6352	IC1257	N6981 (M72)	N7006
Ter 1	Ter 9	IC 1276 (Pal 7)	N6723	N7078 (M15)
Djorg1	N6540	N6637 (M69)	N6809 (M55)	

Figure 3.1.1: List of globular clusters was taken and modified from database (e29).

```
47 Tucanae / NGC 104/ C0021-723
(Updated Janury 2017)

RA: 00:24:05.67 DEC: -72:04:52.6   (J2000)

============================================================
Bytes  Format      Explanation
1-8      A8        Star ID
10-32  A11,1x,A11  Position
34-35    A2        Units for position
                    R0 denotes RA, DEC in the J2000 coordinates
                    R5 denotes RA, DEC in the 1950 coordinates
                    XA denotes X, Y in arcseconds
                    XP denotes X, Y in pixels
37-44   F8.4       Period  (days)
46-51   F6.3       Mean magnitude (or maximum magnitude if "max"
                      is indicated in the remarks column)
53-57   F5.3       Light amplitude (range of variability)
59      A1         Colour for mean magnitude and amplitude
                    e.g.  B, V, R, I, J, K
                    or P (for photographic).
61-65   A5         Type of variable (draft 2006 GCVS classifications)
                      CST denotes non variable stars previously designated
                      as variables
67-80   A19        Notes and Remarks  (f denotes field star)

 "--" or "----" indicates no data available
=========================================================================
ID       Position                     Period <mag>  ampl  C Type  Notes/
         RA/X        Dec/Y     Units                              Remarks
=========================================================================
1        00:24:12.65 -72:06:39.9 R0 221.0    13.15  5.0   V M
2        00:24:18.57 -72:07:59.0 R0 203.0    --     3.4   V M
3        00:25:15.96 -72:03:54.8 R0 192.0    12.63  4.7   V M
4        00:24:00.53 -72:07:26.9 R0 165.0    12.34  2.3   V M?
5        00:25:03.63 -72:09:31.7 R0  50.0    11.80  0.4   V SR    Note
6        00:24:25.68 -72:06:30.0 R0  48.0    11.74  1.0   V SR
7        00:25:20.53 -72:06:40.1 R0  52.0    11.83  0.4   V SR
8        00:24:08.59 -72:03:54.9 R0 155.0    12.01  1.7   V M?
9        00:23:40.53 -72:06:00.0 R0   0.7369 13.672 1.07  V RR0   Note
10       00:24:22.46 -71:53:28.6 R0 ----     13.35  0.50  P L?    Note
11       00:25:09.07 -72:02:16.2 R0 ----     12.03  0.80    L     Note
12       00:28:33.85 -72:10:08.8 R0   0.3714 13.90  0.47  V RR1   f;Note
13       00:22:58.37 -72:06:56.1 R0  40.0    12.36  0.7   V L     +long
14       00:24:06.84 -72:03:38.0 R0 ----     --     --      ?     Note
15       00:25:43.91 -72:06:50.9 R0  38.0    11.9   0.3   V L     Note;Fox
16       00:25:23.18 -72:11:05.3 R0  41.0    11.65  0.2   V L     =LW21
17       00:23:55.66 -71:57:28.6 R0  60.0    11.9   0.1     L     Note;Fox
18       00:25:09.19 -72:02:39.7 R0  83.0    11.67  0.5   V L     Note
19       00:24:14.75 -72:04:44.5 R0  83.0    11.46  0.8   V L     f?;Note
20       00:24:14.47 -72:05:09.2 R0 ----     12.30  0.8   V L     P=232?
21       00:23:50.38 -72:05:50.5 R0 ----     12.41  0.7   V L     P=76?+long
22       00:24:10.02 -72:02:38.2 R0  62.0    11.80  0.25  V L
23       00:24:29.53 -72:09:07.8 R0  52.0    11.77  0.4   V L     +long
24       00:23:47.61 -72:02:49.8 R0 ----     --     --      CST   Note
25       00:23:58.94 -72:02:34.6 R0  44.0    11.96  0.2   V L
26       00:24:07.86 -72:04:31.7 R0 ----     --     --      CST   Note
27       00:24:15.14 -72:04:36.5 R0  69.0    12.11  0.75  V L     +long
```

Figure 3.1.2: Part of the list with variable stars from a globular cluster 47 Tuc (e29).

3.1.1 Age of globular clusters

The age of clusters is also important for the thesis because as we mentioned, we were looking for a possible connection between many parameters, including age of clusters. The age of globular clusters was obtained from the several articles shown in attachment of this thesis. Some data concerning the age of the clusters could not be found or their age was estimated to be more than the age of the universe (13.7 Gyr). It is supposed that the universe is 13.7 Gyr old; for that reason I decided not consider data older than 13 Gyr.

3.1.2 Variable stars in globular clusters

Data from (e14) and (e29) mostly contain variable stars of RR, RR0, RR1, SR, M, E, EA, EB, EC, EW, SXPHE, L and LB type. The other types of variables do not have enough stars for the analysis (they contain units of the order of the stars). The notation of the variable star was acquired from (e31). RR, RR0, and RR1 are the RR Lyrae-star type. They differ in pulsation mode; RR0 pulsate in the fundamental mode, RR1 in the first harmonic mode, and the RR type is not defined. We adopted notation that the RRab type is RR0 and RRc type is RR1. The SR type is a semi-regular variable type with subtypes SRA and SRB. Type M indicates mirids. E, EA, EB, EC, and EW are eclipsing variables. EA denotes Algol type, EB Beta Lyrae and EW W UMA (Ursae Majoris) type. Types E and EC are not specified closer. SXPHE are pulsating variables of SX Phe type. L and LB indicates slow irregular pulsating variables, whereas LB indicates a giant stars.

3.2 Methodology

Data processing for each globular cluster and stars inside clusters was done in Excel, using the Pivot table. The Pivot table is a table of statistics in Microsoft Excel. It helps to summarize the bulk data and group them together in a meaningful way, allowing calculation of averages, sums, and many other statistical functions (e17). I calculated how many and what types of variable stars exist inside each cluster. The Pivot table was also used to calculate the average period (in days) for each variable star type in globular clusters. As a result, one average period for each type of variable stars inside cluster was acquired.

During data processing we found out that the data as period and type of variability were not available for each variable star inside cluster (they were missing/unknown). Some variable stars have two types of variability (e.g., a star have been classified as uncertain type - E/CW variable) or were marked by a question mark, see the line 14 in Fig. 3.1.2. In the case that the type is not further classified, such as RR type, we have decided to omit these data. Thus, 263 variable stars were omitted. Several types of variable stars have very few members, hence, the data could not give us relevant information about the behavior and properties of a particular group. In such cases, the data were not taken into account, thus, we did not make a research with these stars.

After I took all the data about the variable stars inside globular clusters from online databases, I could merge them into one large table, which is shown in the attachment. These data are used in the Analysis, in which I looked for correlations.

3.3 Reliability of the data

The globular clusters are mostly a very dense objects and often the stars inside them can not be well resolved. To be sure that our data are not biased regarding apparent stellar density, tests investigating the number of variable stars with respect to half-light radius and core radius were performed. In other words, the stellar density is affected by the fact that the cluster is closer to us, or it is affected by the real physical properties of cluster (i.e., the cluster has a really high density of stars).

Half-light radius (r_h), also referred to as effective radius, is the area at which half the brightness of the whole cluster is emitted (Binney & Tremaine, 2008). The calculation of r_h is performed from the center of a globular cluster and can be used for the measurement of size of a globular cluster (Madrid et al., 2009). Some sources differ on whether half-light radius depends on the sensitivity of the measuring device. Schneider (2006) argues that r_h is affected by the measuring device because the outer region of a cluster may be below the limit of resolving power. Another source states that this value is independent of the sensitivity of the device and that is the reason why the r_h was introduced e12.

Core radius (r_c) is also defined properly. Madrid et al. (2012) claims that it is a radius at which the apparent surface brightness drops by half. Janes (2000) claims that r_c is a radius at which the brightness drops to half the central value. The last frequently cited definition states, that the core radius measures the degree of concentration of stars in the center of a cluster (e13). Globular clusters usually have small core radii (van den Bergh, 2008). It means that the degree of concentration is quite large. The calculation of r_c is usually in arcmin as well as half-light radius, nonetheless some sources state core radius in pc (Bahcall, 1999; e14).

Although the definition of r_c and r_h are not entirely clear, it is the only way to test the observational bias. I took r_c and r_h from database (e29) and we analysed r_c and r_h depending on the number of variable stars in each cluster (Fig. 3.3.1 and 3.3.2). It is expected, that larger and spatially well resolved globular clusters would contain more known variable stars. That may mean a correlation between number of known variable stars and r_c and/or r_h. The dependences in Fig. 3.3.1 and Fig. 3.3.2 show, that the number of known variable stars is not affected by the (angular) resolution of a telescope (ability to divide two point sources into divided images) and the data are not biased regarding the apparent stellar density. Thus, our statistics are not affected much by the angular size and concentration of the globular cluster.

Data may be also biased by the observation length. The short observation time may give us inaccurate information about the object. In the case of long periodic variables, they can not be detected if we will observe them for a short time. We have no idea whether our data are affected by the time bias, because the length of observation is different for each article, and the databases contain data from many articles.

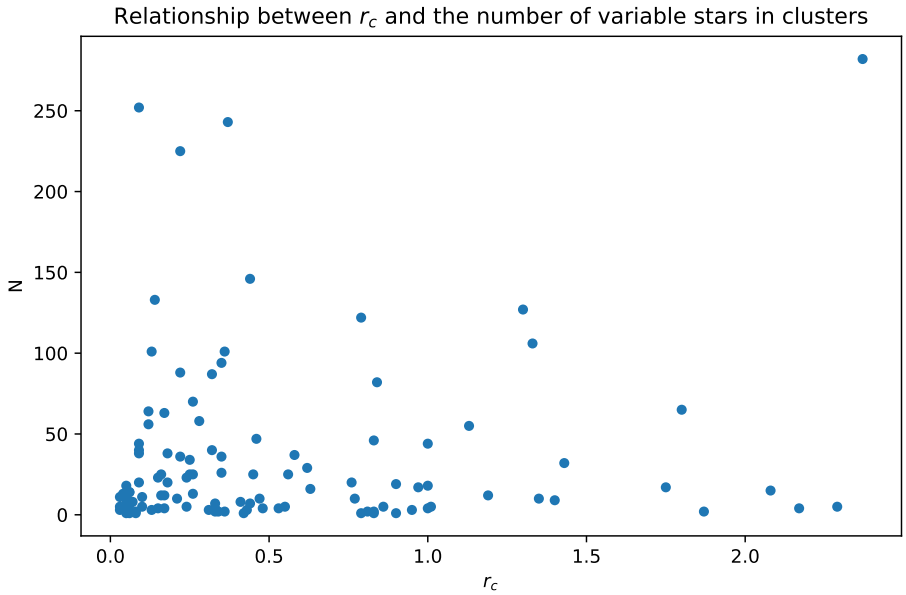

Figure 3.3.1: Dependence of the number of variable stars in globular clusters on the core radius. The variable stars are located in globular clusters with all r_c (g.e., variables are detected in various dense fields.

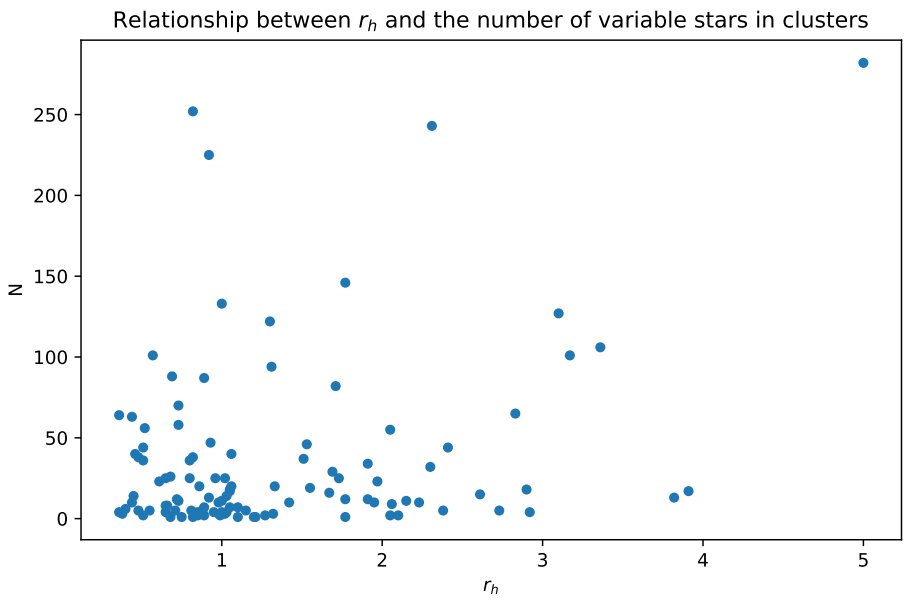

Figure 3.3.2: Dependence of the number of variable stars in globular clusters on the half-light radius. The variable stars are located at all angular distances.

Analysis

Data were processed in Python and used for the analysis. When analysing data, the we focused on the relation between the bimodality of the metallicity distribution of the globular clusters, the age and metallicity of the globular clusters (age-metallicity dependence), the relation between the absolute number of variables of particular type and metallicity of globular clusters, the relation between metallicity of the globular clusters and mean period of particular variability type stars (period-metallicity dependence) and the age dependence of the mean periods. Using graphs and histograms enable detection of correlations of the variable stars in globular clusters.

4.1 Bimodality of the metallicity distribution of the globular clusters

The globular clusters show bimodal metallicity distribution. Zinn (1985) claims that there are two peaks for metallicities -1.6 and -0.5. Smith (2000) also found two peaks but they belongs to metallicities -1.5 and -0.5. Our data in Fig. 4.1.1 tends to the values of metallicites $-1.48(88)$ and $-0.49(41)$ and they are consistent with (Smith, 2000; Zinn, 1985) results. The reason of bimodal metallicity is the diverse time formation of clusters (Muratov & Gnedin, 2010). The later creation of clusters caused that the material was enriched with heavier elements and new stars inside clusters contain more heavier elements. On the other hand, an earlier creation of clusters resulted in metal-poor clusters because the heavier elements were not available. Metal-poor clusters are older than metal-rich clusters and they mainly occur in the halo (e15). Time of the origin reaches back to the formation of the Galaxy. Metal-rich clusters are mostly found in the Galaxy bulge. A possible explanation that the halo is more metal-poor than bulge is that the gas from the halo was expelled before the supernovae could enrich the gas (gas is necessary for the formation of new stars) (e16). This gas was captured in the bulge. This explanation is related to the formation of the Galaxy.

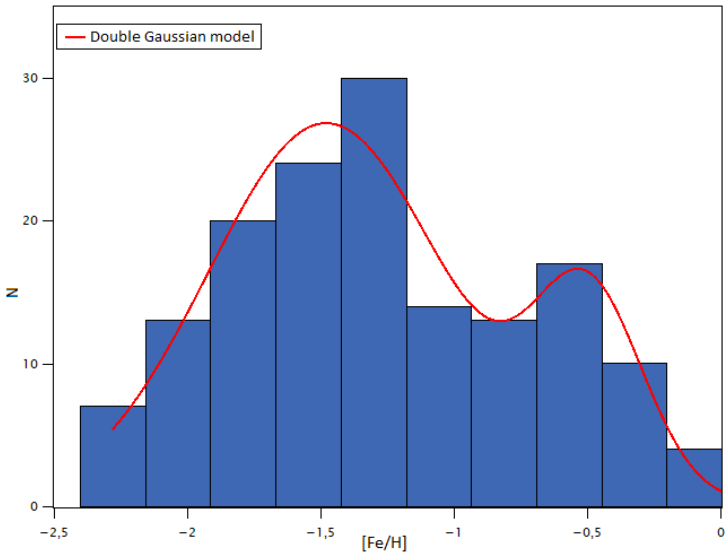

Figure 4.1.1: The graph shows a bimodal metallicity relation. The red line is the fit using Gaussian model corresponds to the bimodal histogram. There are two peaks for metallicities -1.48 and -0.49.

4.2 Ages and metallicities of globular clusters

As we saw in Sect. 4.1, the age and metallicity of a globular cluster is somehow linked. The age-metallicity relation is shown in Fig. 4.2.1. We can see two linear dependencies; one for globular clusters located in the Galactic bulge and one for those located in the Galactic halo. Linear fit for the bulge satisfies the equation 4.3 and for halo we have equation 4.4. The linear dependencies give us some idea of the distribution of halo and bulge clusters for the age-metallicity relation. The both linear functions are decreasing and the functions are almost parallel to each other. The same correlation is mentioned in the article (Carretta et al., 2010), but they did not use any fit to search for dependencies.

$$[\mathrm{Fe/H}] = -0.25(9)Age + 2.0(1.1) \tag{4.3}$$

$$[\mathrm{Fe/H}] = -0.232(32)Age + 1.14(37) \tag{4.4}$$

The membership of clusters in the Galaxy was taken from (Bica et al., 2016). This source shows the globular clusters located in the galactic bulge, so I adopted this list under consideration, that the other globular clusters belong to the halo. It is obvious that metallicity decreases with age (younger cluster – more metal-abundant). Metallicity increases faster in the bulge than in the halo (a halo is younger than a bulge with the same metallicity). In other words, if we have a cluster from bulge and halo with the same age, the bulge has higher metal abundance. In the graph we can see four points featuring young age and high metallicities (Whitting 1 Pal 1, Ter 7, and BH 176). Data shows that the youngest cluster (Whitting 1) is 6.5 Gyr old and it is located in the halo. Models suggest that it was a dwarf galaxy disrupted by the Milky Way (Carraro et al., 2007). Pal 1 and Ter 7 were dwarf galaxies too (Niederste-Ostholt et al., 2010; e19). The origin of BH 176

is unclear; some sources claim that it is an open cluster, not a globular cluster (Davoust et al., 2011).

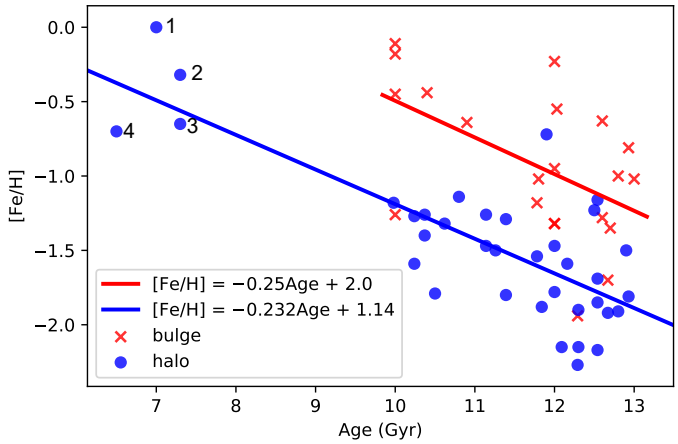

Figure 4.2.1: Age-metallicity relation for globular clusters in Galaxy. Red symbols indicate globular clusters located in the bulge and the blue circles show the globular clusters located in the halo. The circles marked with numbers from one to four are for the globular clusters Whitting 1 (number 4), Pal 1 (3), Ter 7 (2), and BH 176 (1). The continous lines characterize linear regressions and their equations are shown at the bottom left of the graph.

4.3 Occurence rate of the variable types with respect to metallicity

The second topic for discussion is the Occurence rate of the variable types-with respect to metallicity (Occurance-metallicity relation). Our data suggest that the globular clusters are mostly occupied by RR Lyrae stars (52 %), Eclipsing binaries (8 %), SX Phoenicis stars (6 %), Slow irregular variables variables (5 %) and Semi-regular (4 %).

The graphs for the RR0 in Fig. 4.3.1 and the RR1 type in Fig. 4.3.2 are quite similar. The RR0 type is evenly distributed over all metallicity values, but the peak is for metallicity -1.4 (there are the most stars of RR0 type). Metallicity of the stars and clusters depends on the age as we already mention in section 4.2, thus for metallicity value -1.4, the age corresponds to 11 Gyr (the globular clusters located in halo). RR Lyrae are located in the horizontal giant branch so it is clear that these stars are very old and it causes that the most stars of RR0 type in the metal-poor clusters are of the value -1.4. The same peak is for RR1 type. Otherwise the metal-rich globular clusters containing RR1 type stars do not have as many stars as RR0 type. We can estimate, that the possible reason is, that the metal-rich bulge contain more RR0 type stars than RR1 type.

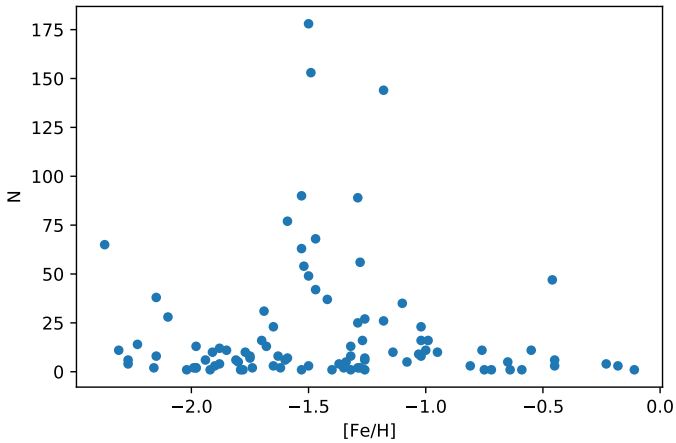

Figure 4.3.1: Occurence rate-metallicity relation for total number of RR0 type stars. The vertical axis shows numbers of RR0 type stars inside each cluster.

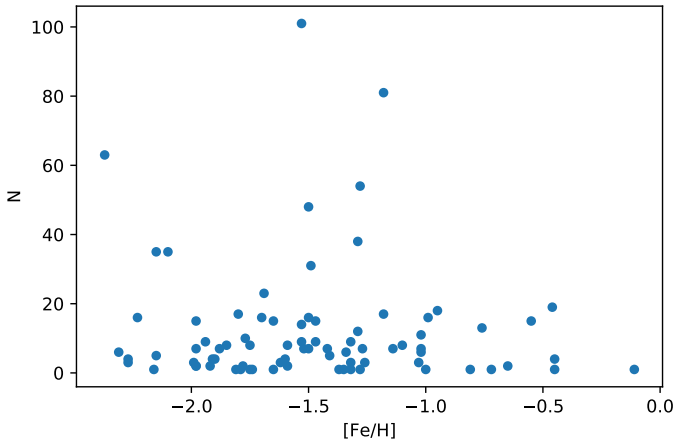

Figure 4.3.2: Occurence rate-metallicity relation for RR1 type stars. The vertical axis shows numbers of RR1 type stars inside each cluster. It is obvious that majority of RR1 type stars are located in metal poor clusters.

The increasing number of SR (semi-regular) variable stars for some metallicity values in the globular clusters is not clear for majority of metallicities, but few metallicities containing more cluster than others. These metallicities are: -1.5, -1.25, and -0.5, see Fig. 4.3.3. Nevertheless, these peaks may be just a coincidence depending on the selection (random) effect and the correlation does not exist. One point (cluster) stands out and belongs to cluster NGC 6715 (point 1 in Fig. 4.3.3). This cluster is located in the Sagittarius dwarf spheroidal galaxy and contains over 40 semi-regular variable stars.

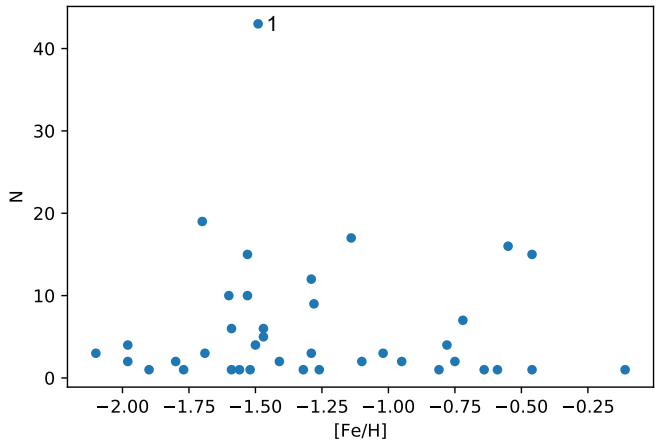

Figure 4.3.3: Occurence rate-metallicity relation for SR type stars. These globular clusters containing SR variables lie in the whole range of metallicities; from -2.2 to -0.2.

SX Phe variable stars are mostly located in globular cluster with whole range of metallicities, see Fig. 4.3.4. The possible explanation why some metal-rich globular clusters containing SX Phe stars and why they are not only distributed in metal-poor clusters is in the article (Nemec et al., 2017). The research of metal-rich SX Phe stars claims, that rotation of SX Phe variables is connected to metallicity (if SX Phe are fast rotators, 2/3 of them are metal-rich stars), however, this does not explain the lack of SX Phe in metal-rich globular clusters. Figure 4.3.4 also shows that five points are outlying. These clusters belong to clusters containing over one hundred variable stars up to 460 stars, in the case of Omega Cen. If these clusters had less stars they may would have similar number of SX Phe stars as the other clusters. But there is one exception, the cluster NGC 6809 contain 37 SX Phe stars and together it has 71 variable stars. We do not have a good explanation why some clusters contain mainly one type of variable stars.

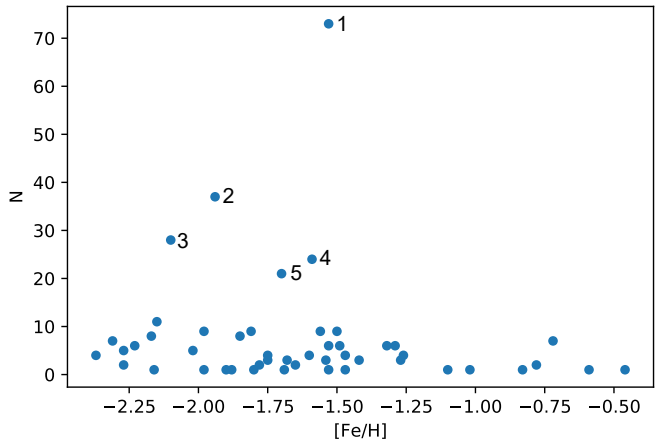

Figure 4.3.4: Occurence rate-metallicity relation for SXPHE stars. Label 1 (NGC 5139) contains 73 variable stars of SXPHE type, this cluster has the most SXPHE variables of all. The five outlying points belong to clusters NGC 6809 (2), ngc 6656 (5), NGC 5139 (1), NGC 5024 (3) and NGC 3201 (4).

The L variables, see Fig. 4.3.5, have the most stars inside clusters in the range of metallicities from -2 to -0.5. The majority of globular clusters contain up to 10 variables of the L type, but five points are outlying. They belong to clusters with many members. Each of these clusters contain from 120 to 350 variable stars.

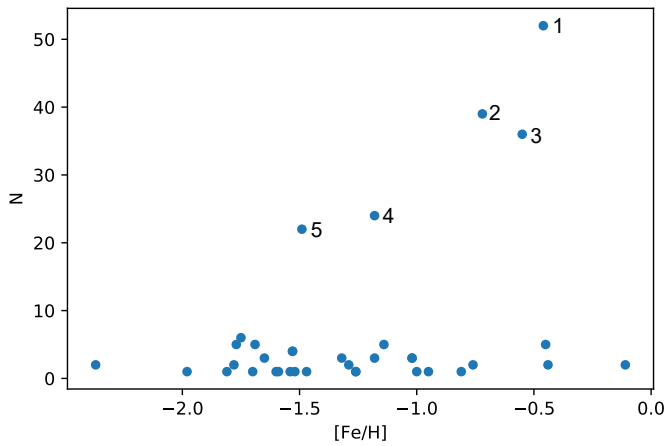

Figure 4.3.5: Occurence rate-metallicity relation of L type. It is obvious that L type is not as metal-poor as SXPHE type. Data in graph contain five outlying clusters, these are NGC 104 (2), NGC 6266 (4), NGC 6388 (3), NGC 6441 (1) and NGC 6715 (5).

4.4 Metallicity dependence of the mean periods

The period-metallicity relation (PMR) for RR0 type is shown in Fig. 4.4.1. The linear fit shows, that the period decreases with increasing metallicity and the period for [Fe/H]=0 is 0.54 days, see the equation 4.5. The same relation they found out in Sandage (1993), although they use LogP. The linear fits works well for RR0 and RR1 variables. The both fits have decreasing character and the fit for RR0 is more steep than for RR1 type.

$$\text{Period} = -0.048(15)[\text{Fe/H}] + 0.542(23) \tag{4.5}$$

RR0 type has three outlying points. They belong to NGC 6388, Terzan 5, NGC 6441 clusters. NGC 6388 and 6441 are metal-rich globular clusters with long period RR Lyrae stars. Terzan 5 is a cluster located in the bulge, thus we can expect metal-abundance, and we can expect the same behavior as NGC 6388 and NGC 6441.

The PMR for RR1 stars is shown in Fig. 4.4.2. The linear regression shows one strong group whose period decreases with metal abundance and the period for [Fe/H]=0 is corresponds to the value 0.28 days, see equation 4.6.

$$\text{Period} = -0.034(8)[\text{Fe/H}] + 0.281(12) \tag{4.6}$$

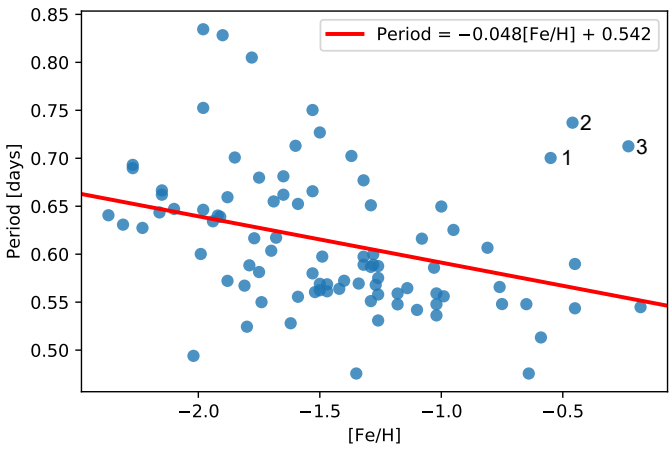

Figure 4.4.1: RR0 period-metallicity relation. The three points are outstanding. Number 1 belongs to NGC 6388, number 2 is NGC 6441 and number 3 belongs to Ter 5.

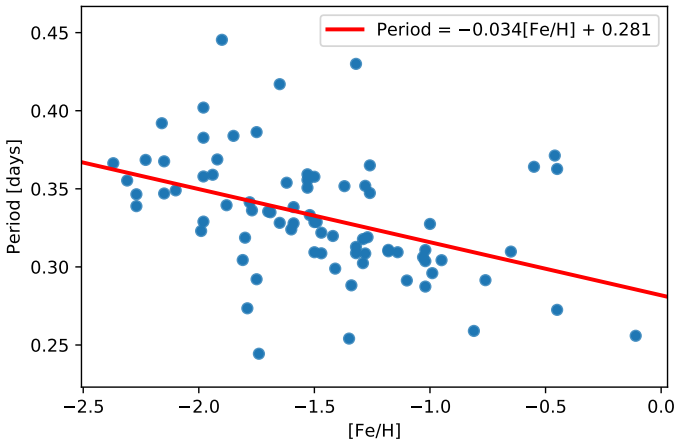

Figure 4.4.2: RR1 period-metallicity relation. The red line is linear fit and has the decreasing character as the linear fit for RR0 type.

The Fig. 4.4.3 shows period-metallicity relation for SXPHE variables. The majority of points (clusters) lie in the metal-poor part of the graph, but a few points lie in the metal-rich part. Thanks to linear regression we could estimate that period increases with metal abundance, see equation 4.7. There are two outlying points with longer periods than the others and belong to NGC 4147 and NGC 6838. The majority of articles studied period-luminosity relation, but one of them were interested in period-metallicity relation and obtained the same results (McNamara, 1995). They also noticed, that the most metal-poor clusters contain SXPHE variables with shorter periods and the metal-rich clusters contain SXPHE with longer periods. They also suggested an explanation; the metal poor clusters evolved from low-mass zero age main sequence.

$$\text{Period} = -0.021(7)[\text{Fe/H}] + 0.096(12) \tag{4.7}$$

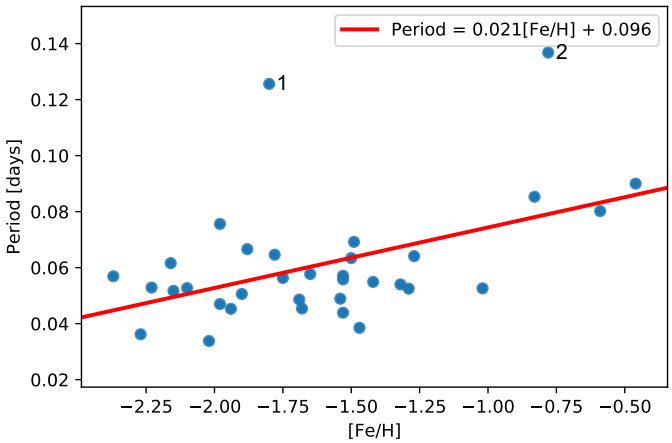

Figure 4.4.3: SXPHE period-metallicity relation. Two outstanding points belong to NGC 4147 (number 1) and NGC 6838 (number 2).

The eclipsing binaries (E) are evenly distributed along all metallicities Fig. 4.4.5. The periods lie between the values of 0.23 days and 3.9 days. However, one point stands out (period = 16.69 days, metallicity = -1.03). This point with a large period belongs to Terzan 1 and can be explained by the fact that the star of E type does not belong to Terzan 1. It is worth mentioning that the EA type is mostly more metal-poor than the EB type, see Fig. 4.4.4. The EA type has metallicity mainly between the values of -2.25 and -1.25, while the EB type has metallicity between the values of -1.5 and -0.5. EA types are detached systems and they do not affected each other. EB types are semi-detached near systems and the mass may transfer to the second component. Thanks to this, the system is evolving faster and the metal-poor systems are already evolved while the metal-poor EA types are still evolving. The EW type is distributed along the whole range of metallicities. EW types are mostly small-mass contact systems so they exist for long time.

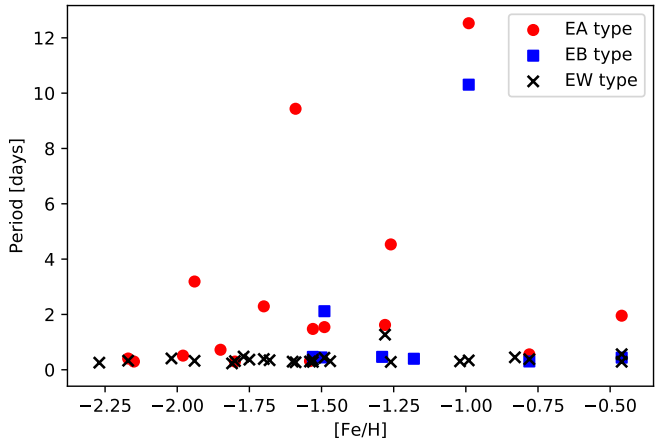

Figure 4.4.4: Period-metallicity relation for EA, EB and EW type. EA type displays a red circles, the EB type has a blue circles and EW type has black symbols.

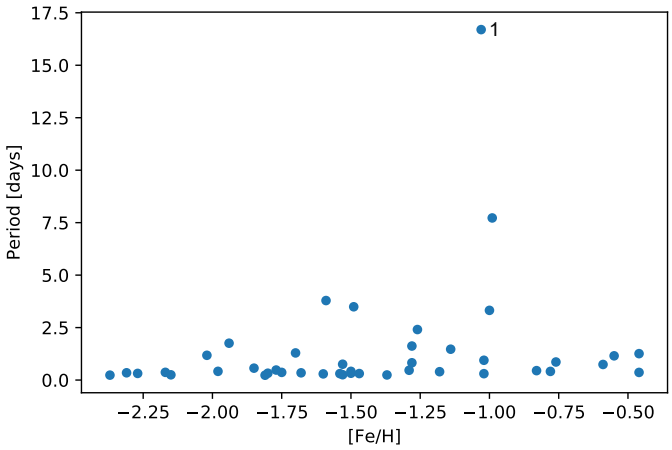

Figure 4.4.5: Period-metallicity relation for E type. This graph displays all types of eclipsing binaries. We took the mean period of E types for each cluster. One point stands out and belongs to Terzan 1 (number 1).

The M type (Mirids) and their PMR is shown in Fig. 4.4.6. There are two groups forming the letter U. The first group has a decreasing character. The period decreases as the metallicity increases. Most points (clusters) belong to the Galactic halo and go from: period = 322 days, metallicity = -2.23; to: period = 180.85 days, metallicity = -1.26. The second group has an increasing character. Period and metallicity increase. The most of these clusters, and the stars of the Mira type inside them, lie in the Galactic bulge. The physical properties of Mira variables are complicated and affected by many parameters. Physical properties are complicated because it is hard to detect high amplitudes in long periodic systems (g.e., Mira variables) which shows no cycles in one night. The radial velocities of Mira are a function of period (e.g., (Feast, 1963)), and period of Mira variables might be a function of metallicity (Feast & Whitelock, 2000). The (Lloyd Evans, 1983) also noticed, that the H-emission spectra are typical for metal-deficient clusters and Mira variables with short periods. The variation of the period versus metallicity apparent from Fig. 4.4.6 might point towards such relations.

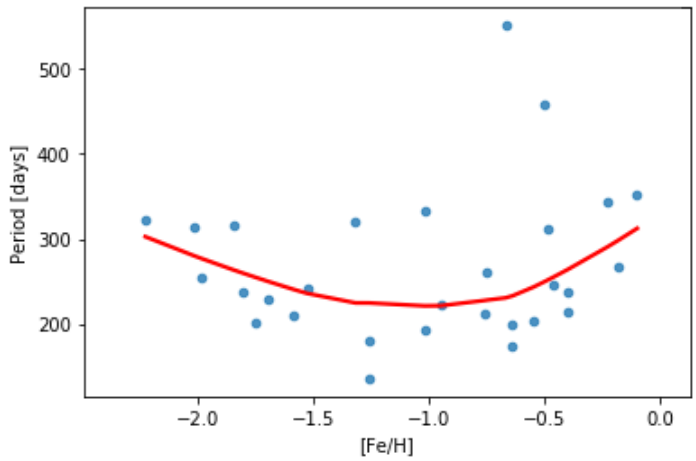

Figure 4.4.6: Period-metallicity relation for M type. Mirids can be found in metal-poor and also in metal-rich clusters.

4.5 Age dependence of the mean periods

As we already discussed in section 4.2, there is a linear dependence between the age of cluster and metallicity. We can expect that there is also age-period relationship. The dependence was obvious for RR0 type, RR1 type and SXPHE type of variable stars. The other types do not show any correlation between age of cluster and mean period of type of the variable stars.

The Fig. 4.5.1 for RR0 type has increasing character; the period increases with the age. The linear regression confirms our statement, see equation 4.8. This conclusion is expected because the Fig. 4.4.1 claims that the period increases with decreasing metallicity, the Fig. 4.2.1 claims, that metallicity decreases with the age. Wherefore, increasing period – older age. On the other, the linear dependence is very weak and it is not as obvious as for RR1 type. RR1 type, see Fig. 4.5.2 and the linear fit satisfies the equation 4.9. This correlation is not mentioned in articles, because this dependence is based on the age-metallicity relationship and period-metallicity relationship, which were studied in many articles.

$$\text{Period} = 0.0238(93)Age + 0.337(108) \tag{4.8}$$

$$\text{Period} = 0.0093(49)Age + 0.2226(58) \tag{4.9}$$

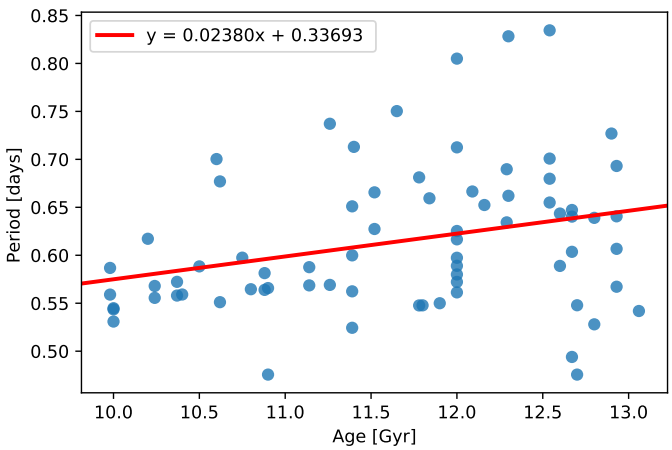

Figure 4.5.1: Age-period relation for RR0 type. The red line shows the linear regression.

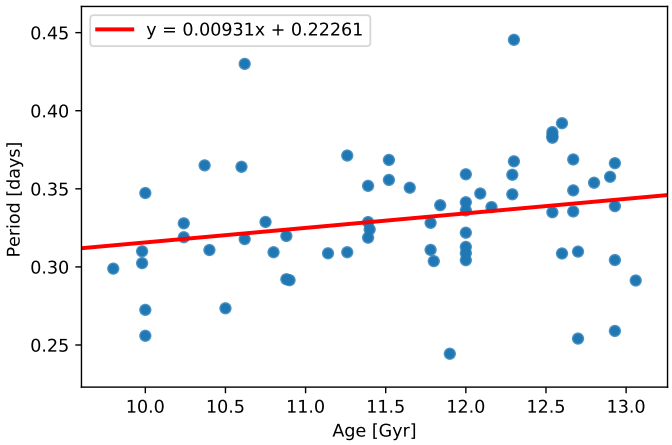

Figure 4.5.2: Age-period relation for RR1 type. The linear regression is similar to RR0 type.

SXPHE type works in the opposite diretion, see Fig. 4.5.3. There we can see that the period decreases with age and the linear fit satisfies the equation 4.10. The article (e18) studies the same relationship, although they use LogP instead of period. They get different relationship because they fit the cubic polynomial, but we use the linear fit/polynomial. We have no reason to fit a polynomial, because the data shows no other than linear dependence. It may be caused by the fact, that they took data for SXPHE and Delta Scuti variables. Our data contain only SXPHE variables.

$$\text{Period} = -0.0033(44)Age + 0.097(51) \tag{4.10}$$

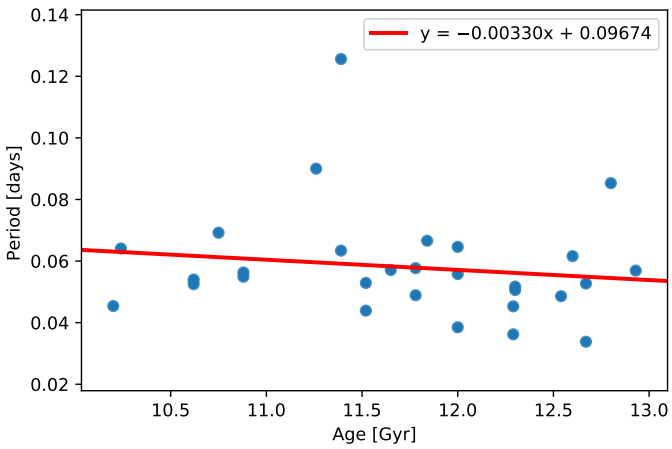

Figure 4.5.3: Age-period relation for SXPHE type. The red line shows the linear regression and has the decreasing character.

Discussion and summary

The aim of the thesis was to reveal the dependences between types of variable stars inside globular clusters and between individual properties of globular clusters such as metallicity and age. We took a list of globular clusters and variable stars from (e14), (e29) and processed them in Python and in Excel, using a pivot table. The age of clusters was taken from many articles, see the electronic table attached to this thesis, and the location of clusters was taken from (Bica et al., 2016). We also investigated whether our data are biased regarding apparent stellar density using half-light radius and core radius. It was found that the data are not affected much by the angular size and by the concentration of the globular cluster.

First, we analyzed bimodality of the metallicity distribution of the globular clusters. The bimodality is known for the globular clusters, so we did not expect a different result. We fit the data by using two Gaussian models and there are two peaks for metallicities $-1.48(88)$ and $-0.49(41)$. Globular clusters contain old stars which correspond to low metallicities. Metal-rich clusters (i.e., the stars inside them) may have been formed later during a later merging of Galaxy.

The metallicity of a cluster is affected by its location in the Galaxy. We also found the age-metallicity relation which shows two linear dependences. It is obvious that metallicity decreases with age (the younger the cluster – the higher the metal abundance) in case of galactic halo and bulge. The clusters inside the Galactic bulge are of about 0.86 dex more metal-rich than the clusters inside the halo of the same age. Halo was formed during the formation of the Galaxy from unenriched gas and from absorbing dwarf galaxies by our Galaxy. Metal-rich halo clusters may have arisen by supernovae explosion, where the explosion could have caused gas enrichment.

The research of occurrence rate of the variable types with respect to metallicity shows a rough idea of the distribution of the variable types in globular clusters. Data confirm that globular clusters are mostly occupied by RR Lyrae stars (52 %), Eclipsing binaries (8 %), SX Phoenicis stars (6 %), Slow irregular variables (5 %) and Semi-regular (4 %). The majority of them are formed by RR Lyrae stars which contain the whole range of metallicities.

The research of metallicity dependence of the mean periods shows in some cases linear dependence. For the RR Lyrae stars, the period of pulsations decreases in metal-rich stars/clusters. This dependence applies to both RR0 and RR1 variables with the difference that the RR1 type have shorter periods than the RR0 type.

The variables of SXPHE stars show linear dependence similar to the RR Lyrae variables but the period increases with increasing metallicity. The explanation of this dependence is not clear, however, this result is supported by the article (McNamara, 1995).

If we do not distinguish between different types of eclipsing binaries, then they are distributed across all values of metallicity with a mean periods up to 8 days. There is one exception, Terzan 1, the globular cluster located in the Galactic bulge. The stars inside Terzan 1 have mean periods 16.7 days long. These E variables may not be the members of cluster and they harm the data. If we plot data of EA, EB, and EW types separately, we can notice that the EA type is located in the metal-poor part of the graph while the EB type is in the metal-rich part of the graph. The EB types are a semi-detached systems, where the mass may transferred between the components, and as a result, the system evolves faster. Consequently, the first-born metal-poor EB type variables complete their evolution, while the detached EA types are still evolving. The EW type is located in the whole range of metallicities because the small masses of EW type components allow the existence of their stars for a long time.

Mirids are long-period variable stars. Up to the metallicity value of -1.3, the period decreases with increasing metal abundance. Above the value of -1.3, the period increases with rising metal abundance. These stars are not well explored because their long periods need long observation time, thus we do not have enough information about them. The large number of Mira variables are located in the metal diverse bulge (Feast & Whitelock, 2014). We can expect than the metal-poor Mira variables can be found primarily in the Galactic halo.

The last topic of my thesis is the age dependence of the mean periods. This investigation clearly showed and connected the age-metallicity dependence and period-metallicity dependence. The age dependence of the mean period research was performed on the RR Lyrae and SX Phoenicis variables. These variables show linear dependence in the period-metallicity relation and the age-metallicity relation shows the linear dependence for clusters too; therefore, we expect the same correlation for the age-period dependence. The period increases with the age for RR Lyrae variables. This correlation is not mentioned in articles, because this dependence is based on the already mentioned age-metallicity relationship and period-metallicity relationship. SXPHE variables show that period decreases with age. It is the opposite phenomenon to the one exhibited RR Lyrae. This dependence is also in correlation with age-metallicity dependence and period-metallicity dependence.

It will be worth extending this thesis to a mass-metallicity relation and to compare properties of variable stars inside globular cluster with variable stars in the whole Galaxy.

References

Sources

Bahcall, N. A. 1999, Formation of Structure in the Universe, 135

Bailey, S. I. 1902, Annals of Harvard College Observatory, 38, 1

Bica, E., Ortolani, S., & Barbuy, B. 2016, , 33, e028. doi:10.1017/pasa.2015.47

Binney, J. & Tremaine, S. 2008, Galactic Dynamics: Second Edition, by James Binney and Scott Tremaine. ISBN 978-0-691-13026-2 (HB). Published by Princeton University Press, Princeton, NJ USA, 2008.

Blažko, S. 1907, Astronomische Nachrichten, 175, 325. doi:10.1002/asna.19071752002

Carraro, G., Zinn, R., & Moni Bidin, C. 2007, , 466, 181. doi:10.1051/0004-6361:20066825

Carretta, E., Bragaglia, A., Gratton, R. G., et al. 2010, , 516, A55. doi:10.1051/0004-6361/200913451

Clement, C. M., Muzzin, A., Dufton, Q., et al. 2001, , 122, 2587. doi:10.1086/323719

Clement, C. 2017, European Physical Journal Web of Conferences, 152, 01021. doi:10.1051/epjconf/201715201021

Cousins, A. W. J. 1976, , 81, 25

Davoust, E., Sharina, M. E., & Donzelli, C. J. 2011, , 528, A70. doi:10.1051/0004-6361/201015514

Dolinsky, J. 2017, *Analýza světelné křivky proměnné hvězdy typu RR Lyrae*, Masarykova univerzita, bakalářská práce

D'Souza, R. & Rix, H.-W. 2013, , 429, 1887. doi:10.1093/mnras/sts426

Feast, M. W. 1963, , 125, 367. doi:10.1093/mnras/125.5.367

Feast, M. W. & Whitelock, P. A. 2000, , 317, 460. doi:10.1046/j.1365-8711.2000.03629.x

Feast, M. & Whitelock, P. A. 2014, Setting the scene for Gaia and LAMOST, 298, 40. doi:10.1017/S1743921313006182

Fukugita, M., Ichikawa, T., Gunn, J. E., et al. 1996, , 111, 1748. doi:10.1086/117915

Harrison, T. E. & Campbell, R. K. 2018, , 474, 1572. doi:10.1093/mnras/stx2881

Hessels, J. W. T., Ransom, S. M., Stairs, I. H., et al. 2006, Science, 311, 1901. doi:10.1126/science.1123430

Janes, K. 2000, Encyclopedia of Astronomy and Astrophysics, 1871. doi:10.1888/0333750888/1871

Jeon, Y.-B., Lee, M. G., Kim, S.-L., et al. 2004, , 128, 287. doi:10.1086/421735

Johnson, H. L. & Morgan, W. W. 1953, , 117, 313. doi:10.1086/145697

Kilston, S. 1975, , 87, 189. doi:10.1086/129744

Kolláth, Z. 2018, The RR Lyrae 2017 Conference. Revival of the Classical Pulsators: from Galactic Structure to Stellar Interior Diagnostics, 6, 137

Kopal, Z. 1955, Annales d'Astrophysique, 18, 379

Leonard, P. J. T. 1989, , 98, 217. doi:10.1086/115138

Lloyd Evans, T. 1983, , 204, 961. doi:10.1093/mnras/204.3.961

Madrid, J. P., Hurley, J. R., & Sippel, A. C. 2012, , 756, 167. doi:10.1088/0004-637X/756/2/167

Madrid, J. P., Harris, W. E., Blakeslee, J. P., et al. 2009, , 705, 237. doi:10.1088/0004-637X/705/1/237

McNamara, D. H. 1995, , 109, 1751. doi:10.1086/117405

Muratov, A. L. & Gnedin, O. Y. 2010, , 718, 1266. doi:10.1088/0004-637X/718/2/1266

Nemec, J. M., Balona, L. A., Murphy, S. J., et al. 2017, , 466, 1290. doi:10.1093/mnras/stw3072

Niederste-Ostholt, M., Belokurov, V., Evans, N. W., et al. 2010, , 408, L66. doi:10.1111/j.1745-3933.2010.00931.x

Salaris, M., Weiss, A., & Percival, S. M. 2004, , 414, 163. doi:10.1051/0004-6361:20031578

Sandage, A. 1993, , 106, 687. doi:10.1086/116675

Schneider, P. 2006, Einführung in die extragalaktische Astronomie und Kosmologie / Peter Schneider. Berlin: Springer, ISBN 3-540-25832-9, ISBN 978-3-540-25832-2, 2006, XV+452 pp.

Schwarzschild, M. 1940, Harvard College Observatory Circular, 437

Sipahi, E. 2012, , 17, 377. doi:10.1016/j.newast.2011.09.003

Smith, G. H. 2000, , 112, 12. doi:10.1086/316498

Strömgren, B. 1956, Vistas in Astronomy, 2, 1336. doi:10.1016/0083-6656(56)90060-5

van den Bergh, S. 2008, , 385, L20. doi:10.1111/j.1745-3933.2008.00424.x

Vaz, L. P. R. 1985, , 113, 349. doi:10.1007/BF00650970

Wallerstein, G. 2002, , 114, 689. doi:10.1086/341698

Watson, C. L., Henden, A. A., & Price, A. 2007, Journal of the American Association of Variable Star Observers (JAAVSO), 35, 414

Zinn, R. 1985, , 293, 424. doi:10.1086/163249

Electronic sources

[e1] https://is.muni.cz/www/limu/trans/navody/PHV.pdf

[e2] https://astro.physics.muni.cz/download/documents/slides/uct_cz-Zejda_Hvězdokupy.pdf

[e3] https://www.cosmos.esa.int/web/hipparcos/variable-stars

[e4] https://sci.esa.int/gaia

[e5] http://www.astro.ubc.ca/MOST/index.html

[e6] https://www.nasa.gov/mission_pages/kepler/main/index.html

[e7] https://www.physics.mcmaster.ca/Fac_Harris/mwgc.ref

[e8] https://www.oxfordreference.com/view/10.1093/oi/authority.20110803125136892

[e9] https://www.swpc.noaa.gov/sites/default/files/images/u33/Chapter_3.pdf

[e10] http://astro.physics.uiowa.edu/ITU/labs/foundational-labs/introduction-to-spectroscop-2/part-5-spectrum-of-the-sun.html

[e11] https://selfstudypoint.in/equipotential-surfaces/

[e12]https://astronomy.swin.edu.au/cosmos/h/Half-light+Radius

[e13]https://www.britannica.com/place/Milky-Way-Galaxy#ref559083

[e14]http://physwww.mcmaster.ca/~harris/mwgc.dat

[e15]https://lss.fnal.gov/archive/other/eso-1049b.pdf

[e16]https://sites.astro.caltech.edu/~george/ay20/eaa-galhalo.pdf

[e17] https://excelpivot.com/

[e18] https://www.stsci.edu/~webdocs/IAU258/39.McNamara.pdf

[e19] https://www.nasa.gov/content/goddard/hubble-looks-into-terzan-7/#.YLNuk6gzaHs

[e20]http://www.thestargarden.co.uk/Spectral-lines.html

[e21]http://mobile.chandra.harvard.edu/edu/formal/variable_stars/bg_info.html

[e22]http://www.ifa.hawaii.edu/users/kud/teaching_17/6_DEB.pdf

[e23]http://stuartmumford.uk/asteroseismology.html

[e24]https://apod.nasa.gov/apod/ap170711.html

[e25]https://www.e-education.psu.edu/astro801/content/l7_p5.html

[e26]https://astrobites.org/2018/11/27/cluster-creation-in-a-cosmological-context/

[e27]https://www.astro.zcu.cz/cs/clanky/promenne/10/

[e28]https://vs-compas.belastro.net/bulletin/issue/2/p6

[e29]http://www.astro.utoronto.ca/~cclement/cat/listngc.html

[e30]https://www.physics.muni.cz/~zejda/zdroje_dat2019.pdf

[e31] https://www.assa.org.au/resources/variable-stars/classifying-variable-stars/