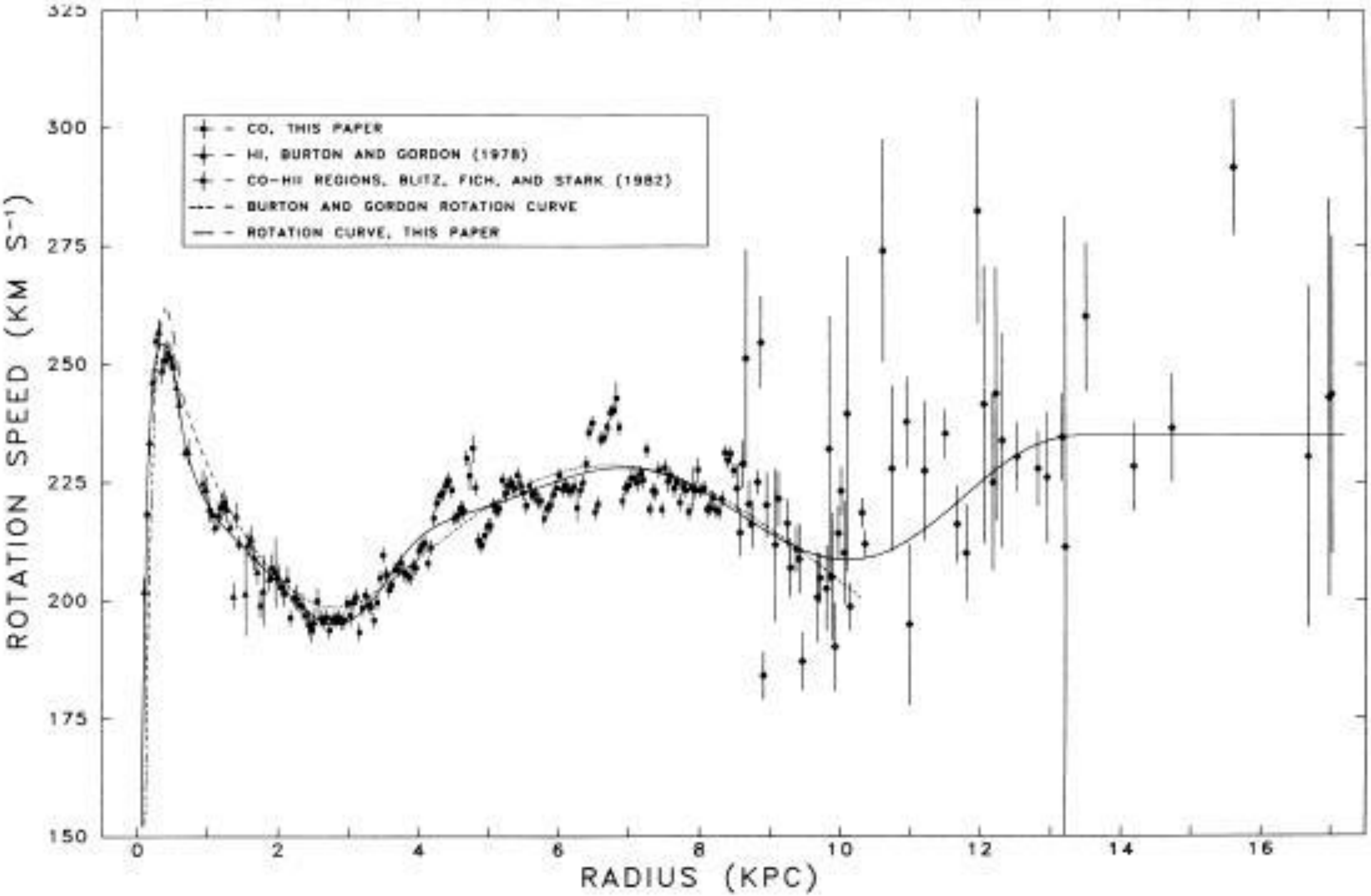
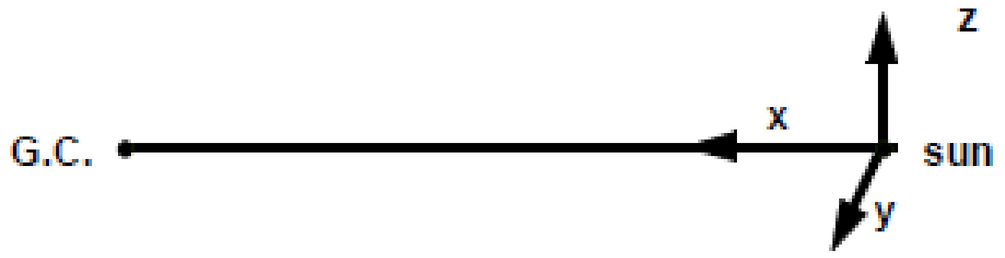
Kinematical membership criteria

- Members follow the motion of the cluster center of gravity
- Internal velocity distribution
- From best to …
  1. Radial velocity and proper motion
  2. Radial velocity
  3. Proper motion



Clemens, 1985, ApJ, 295, 422

Galactic Components



- **coordinates:**
  - x: l = 0 b = 0 (Galactic Centre)
  - y: l = 90 b = 0 (Cygnus) direction of disc rotation
  - z: b = 90 (North Galactic Pole)

- **star positions:**
  - x = r cos b cos l
  - y = r cos b sin l
  - z = r sin b

$$r = \sqrt{x^2 + y^2 + z^2}$$

distance from sun to star

- **star velocities:** $u, v, w \equiv \dot{x}, \dot{y}, \dot{z}$

stellar velocities

- **star velocities:** $$(u,v,w)\equiv\frac{d}{dt}(x,y,z)=(\dot{x},\dot{y},\dot{z})$$

$$u=\dot{x}=\dot{r}\cos b\cos l+r\left(-\dot{b}\sin b\right)\cos l+r\cos b\left(-\dot{l}\sin l\right)$$

$$v=\dot{y}=\dot{r}\cos b\sin l+r\left(-\dot{b}\sin b\right)\sin l+r\cos b\dot{l}\cos l$$

$$w=\dot{z}=\dot{r}\sin b+r\dot{b}\cos b$$

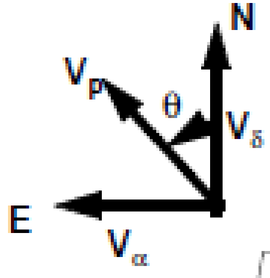
- **Note:**
  - radial velocity $\dot{r}=v_r;$ velocity components along l and b, $[\cos b]r\dot{l}=v_l;$ $r\dot{b}=v_b$

$$u=v_r\cos b\cos l-v_b\sin b\cos l-v_l\sin l$$

$$v=v_r\cos b\sin l-v_b\sin b\sin l+v_l\cos l$$

$$w=v_r\sin b+v_b\cos b$$

motions on plane of sky



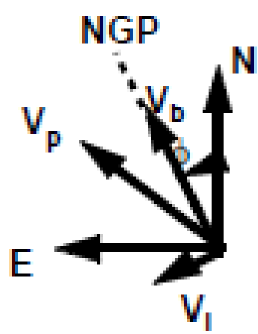
$$v_\delta = r\mu_\delta$$

$$v_\alpha = r\mu_\alpha$$

$$v_p = \sqrt{v_\alpha^{\ 2} + v_\delta^{\ 2}}; \qquad \tan\theta = {v_\alpha}/{v_\delta}$$

- θ : position angle of proper motion (from N to E)

- **in Gal coords:**



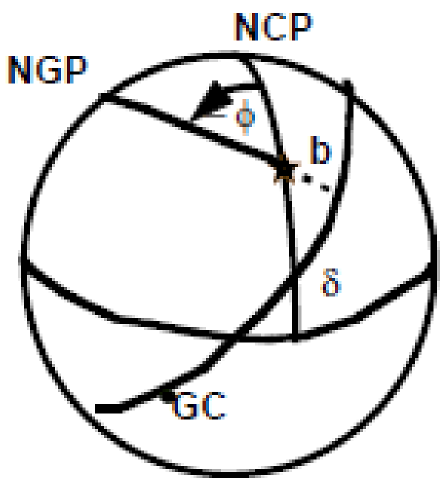
ϕ : position angle of NGP

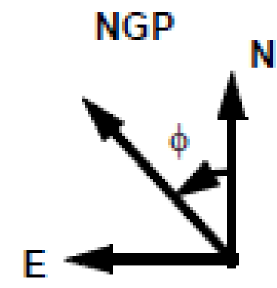
$$v_l = v_\alpha \cos\phi - v_\delta \sin\phi$$

$$v_b = v_\alpha \sin\phi + v_\delta \cos\phi$$

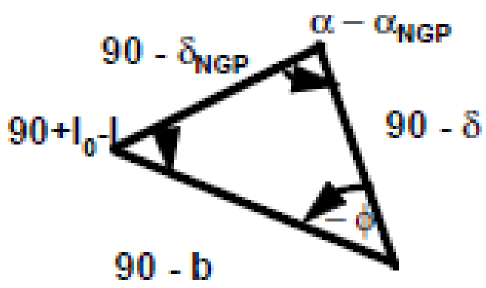
$$v_p = \sqrt{v_l^{\ 2} + v_b^{\ 2}}; \qquad \tan(\theta - \phi) = {v_\alpha}/{v_\delta}$$

Galactic Parallactic Angle





ϕ = position angle (from N to E) of NGP at the star



$$\cos\phi = \frac{\sin\delta_{NGP} - \sin\delta\sin b}{\cos\delta\cos b}$$

$$\sin\phi = \frac{-\sin(\alpha - \alpha_{NGP})\cos\delta_{NGP}}{\cos b}$$

Summary

- **Observe:**

$$\alpha, \delta, r, v_r, \mu_\alpha, \mu_\delta$$

- **Calculate:**

$$\alpha, \delta \Rightarrow l, b$$

$$\alpha, \delta, b \Rightarrow \phi$$

$$r, \mu_\alpha, \mu_\delta \Rightarrow v_\alpha, v_\delta$$

$$v_\alpha, v_\delta, \phi \Rightarrow v_l, v_b$$

$$v_r, v_l, v_b, b, l \Rightarrow u, v, w$$

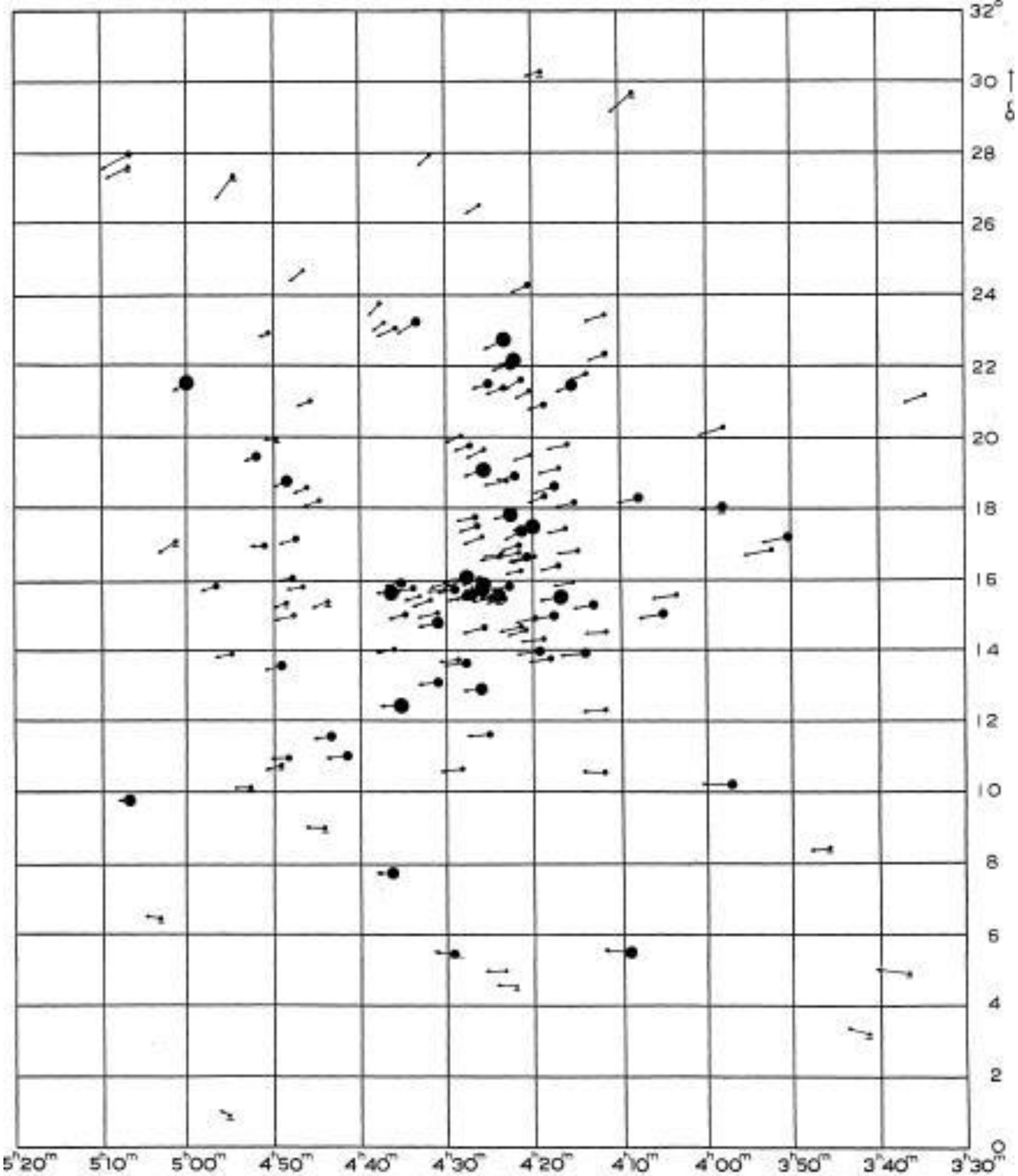
$$r, l, b \Rightarrow x, y, z$$

Motion of the Sun

- Peculiar Apex motion: α = 18h28m, $\delta = +30^{o}$; $l = 56.24^{o}$, $b = +22.54^{o}$
- (U,V,W) = (-10, +5, +7) km s^{-1}
- v_{orbit} = 220 km s^{-1}
- Local Standard of Rest (LSR)

Hyades

Van Bueren, 1952, BAN, 11, 385



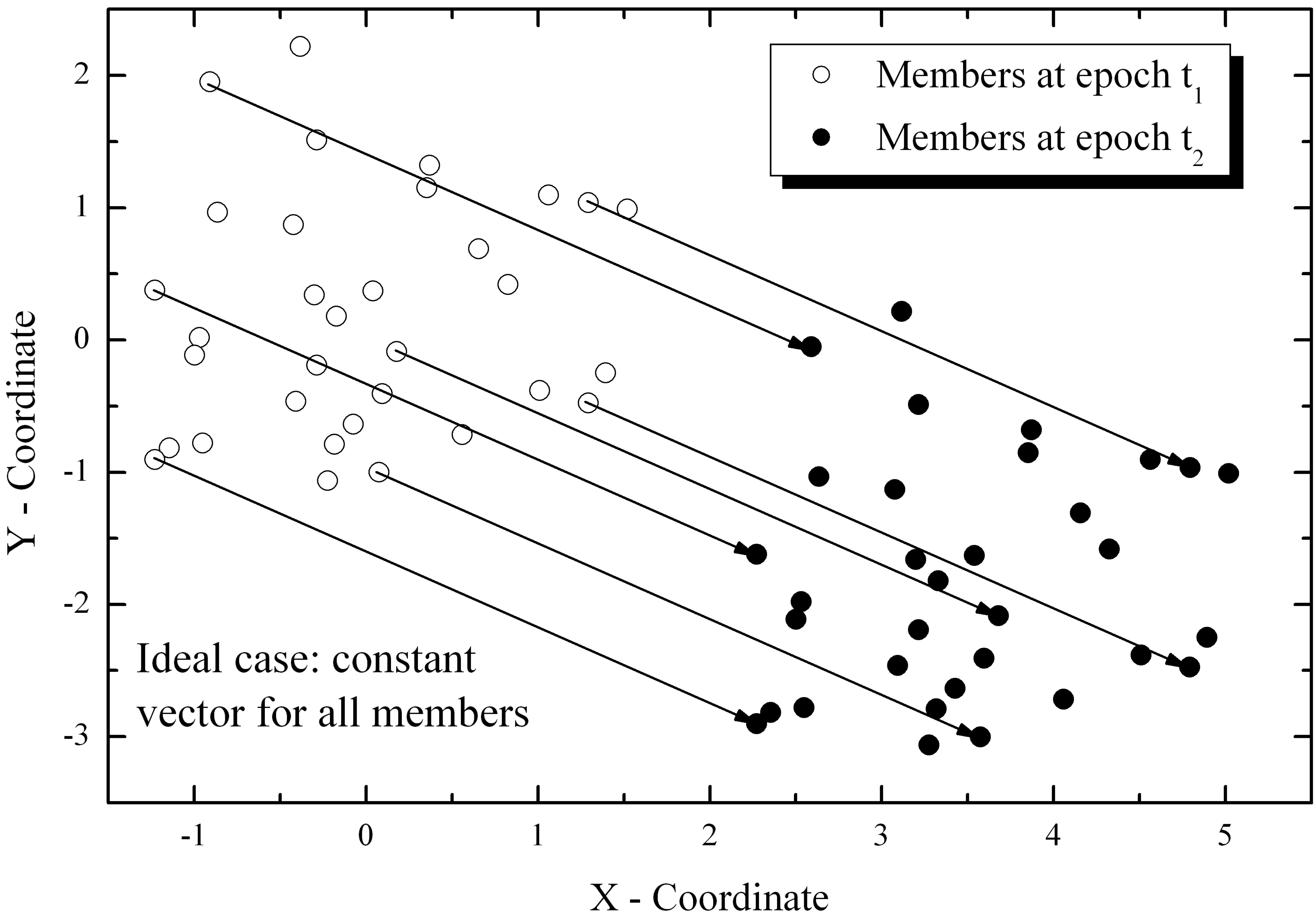
Map of the Hyades brighter than $9^m.0$ visual. The size of the dots is a measure for the magnitude of the stars; the arrows show the annual proper motion (1 mm ≈ $0''/^a.030$). Five stars in the very outer regions of the cluster are not shown on the map. Underlined dots indicate stars of which no radial velocity is known.

After the correction of the solar motion

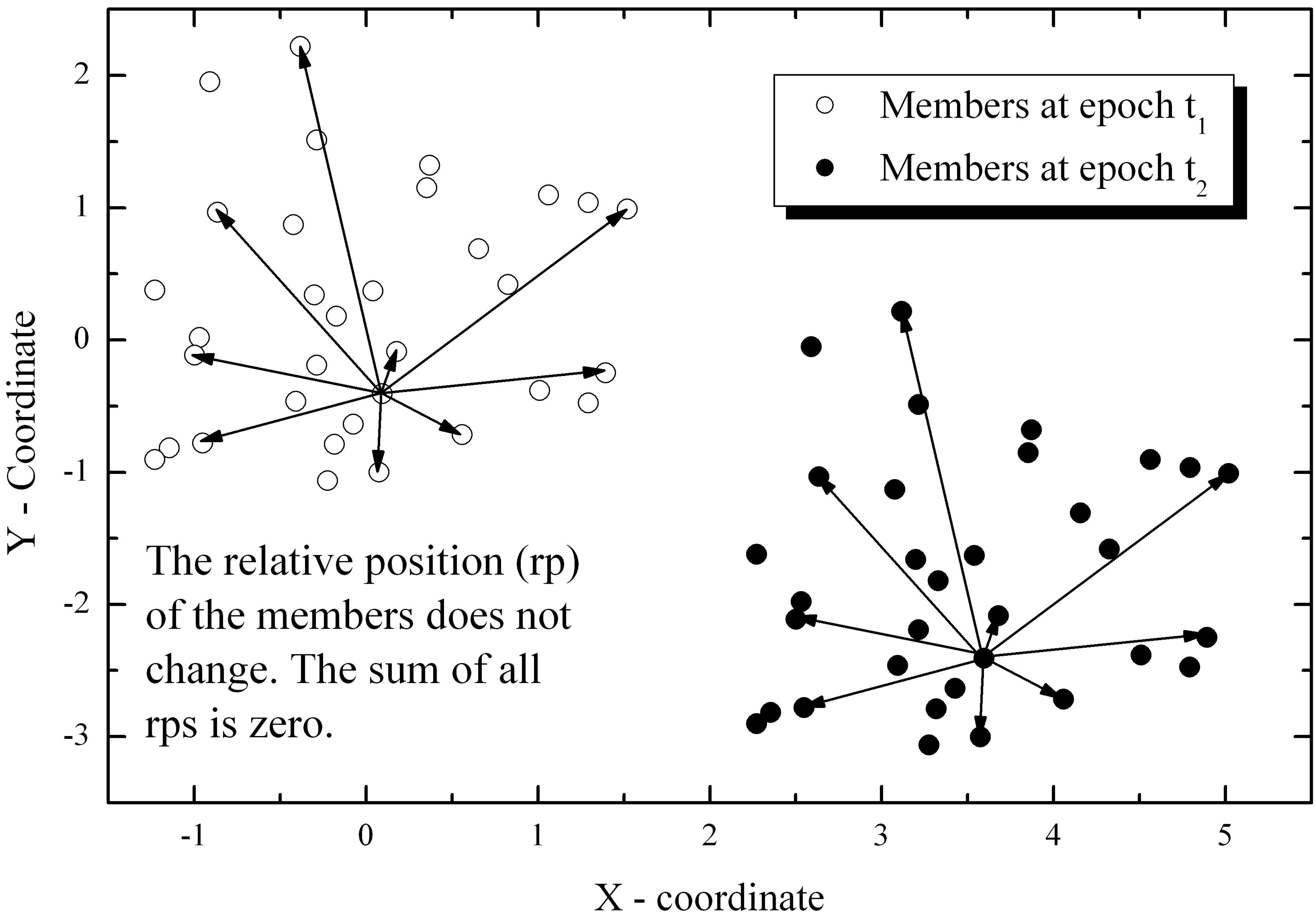
Determination of the kinematical membership

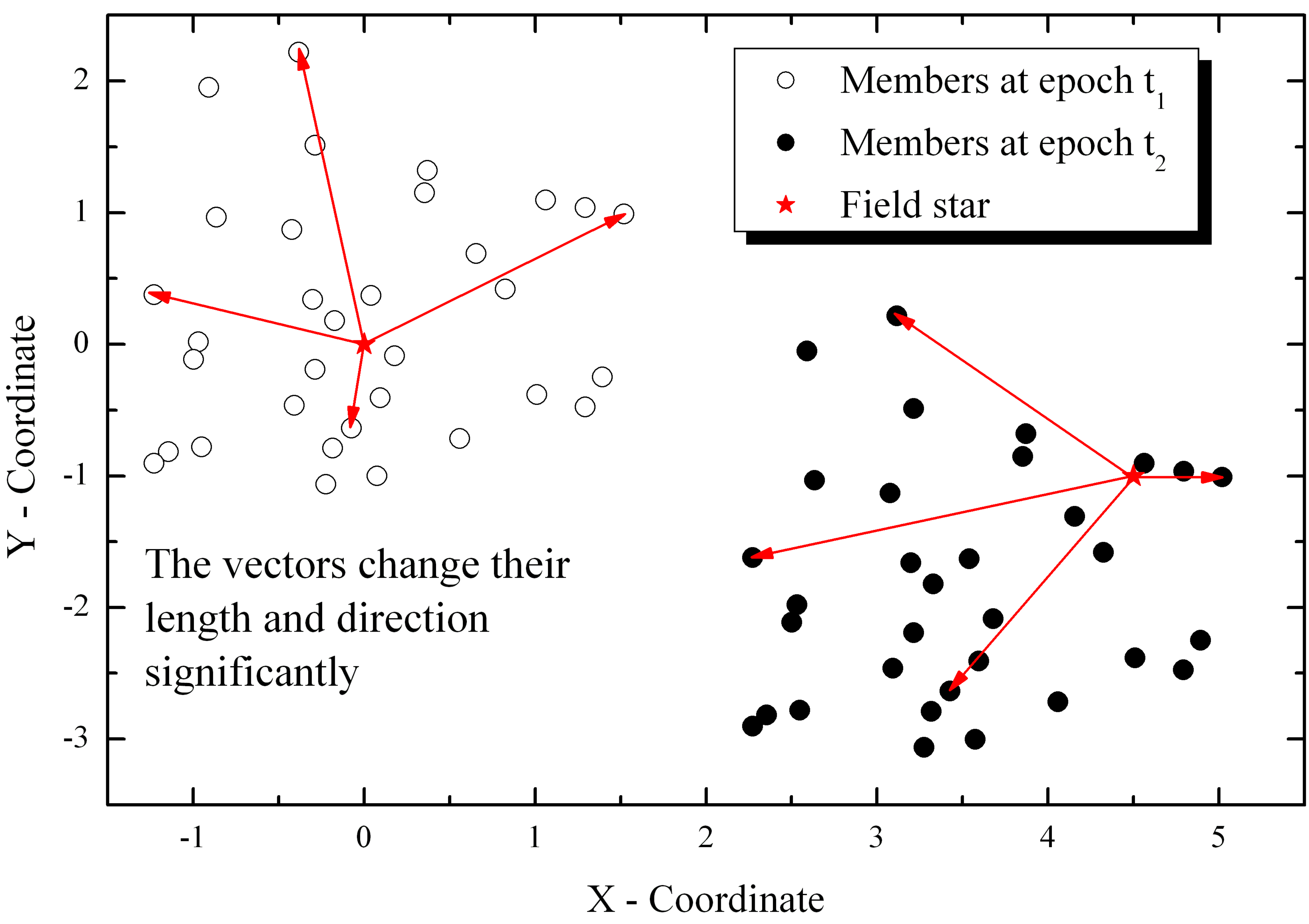
- Three possibilities:

1. Observation of the position at two difference times (= epochs), with a very large time basis. First photographic plates around 1860, largest time scale about 150 years
2. Proper motions of stars in the direction of the Declination α and Right Ascension δ
3. Radial velocity measurements



Members at epoch t1
Members at epoch t2
Ideal case: constant
vector for all members
Y - Coordinate
X - Coordinate
2
1
0
-1
-2
-3
-1
0
1
2
3
4
5


Members at epoch t_1
Members at epoch t_2
The relative position (rp) of the members does not change. The sum of all rps is zero.
Y - Coordinate
X - coordinate
2
1
0
-1
-2
-3
-1
0
1
2
3
4
5



Members at epoch t_1
Members at epoch t_2
Field star
The vectors change their length and direction significantly
Y - Coordinate
X - Coordinate
-1
0
1
2
3
4
5
2
1
0
-1
-2
-3

Mathematical method

- Measurement of the position (X, Y) at two different epochs t_1 (´) and t_2 (´´) for each star
- Calculate the absolute distance in X and Y for both epochs and each star individually

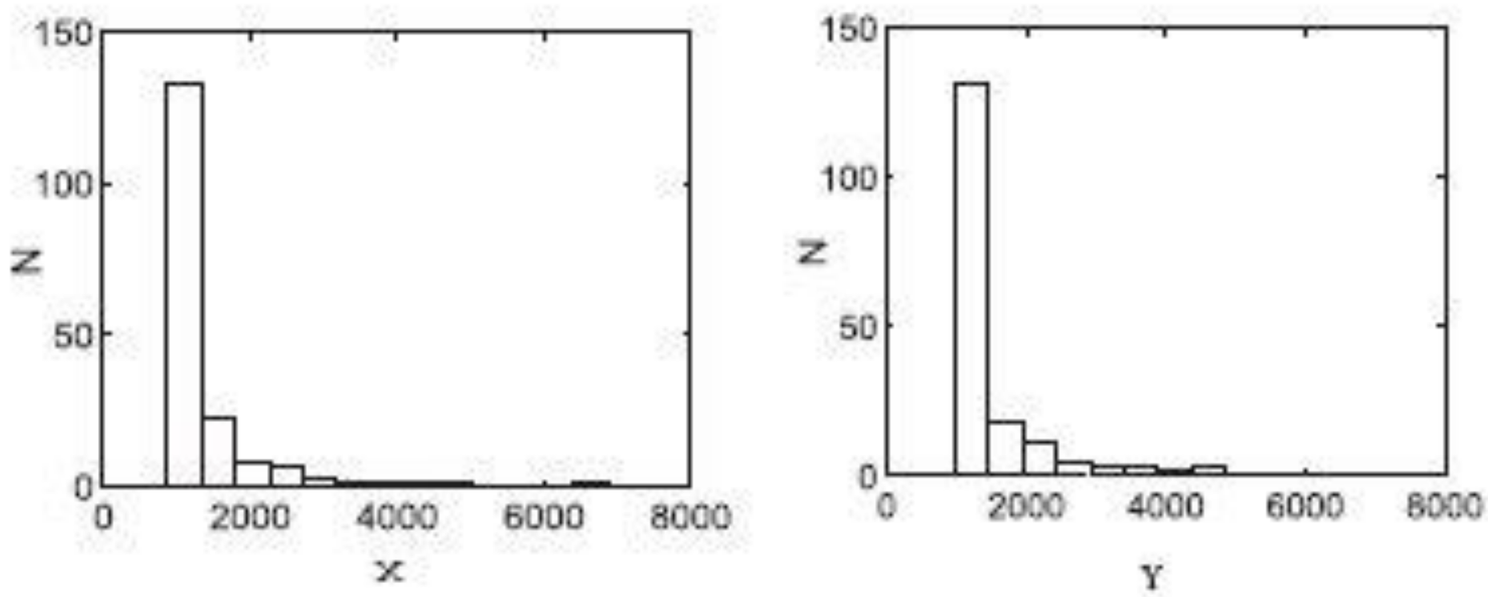
$$S'_{x_i} = \sum_{j=1}^{N} |x'_i - x'_j|, \qquad S'_{y_i} = \sum_{j=1}^{N} |y'_i - y'_j|, \qquad (1)$$

$$S''_{x_i} = \sum_{j=1}^{N} |x''_i - x''_j|, \qquad S''_{y_i} = \sum_{j=1}^{N} |y''_i - y''_j|, \qquad (2)$$

- Determine the differences of the absolute distances

$$\delta S_{x_i} = S'_{x_i} - S''_{x_i}, \qquad \delta S_{y_i} = S'_{y_i} - S''_{y_i}, \qquad i = 1, \ldots, N. \qquad (3)$$

- Plot the histograms of the differences of the absolute distances. The members have to group around the minimum of the distributions (ideal case: minimum = zero).



Example from Javakhishvili et al., 2006, A&A, 447, 915 for Collinder 121

Now we need a mathematical formalism to describe the membership probability from the distributions

- Calculate the absolute distance in X and Y for both epochs and each star individually

$$S'_{x_i} = \sum_{j=1}^{N} (x'_i - x'_j), \qquad S'_{y_i} = \sum_{j=1}^{N} (y'_i - y'_j), \qquad (4)$$

$$S''_{x_i} = \sum_{j=1}^{N} (x''_i - x''_j), \qquad S''_{y_i} = \sum_{j=1}^{N} (y''_i - y''_j). \qquad (5)$$

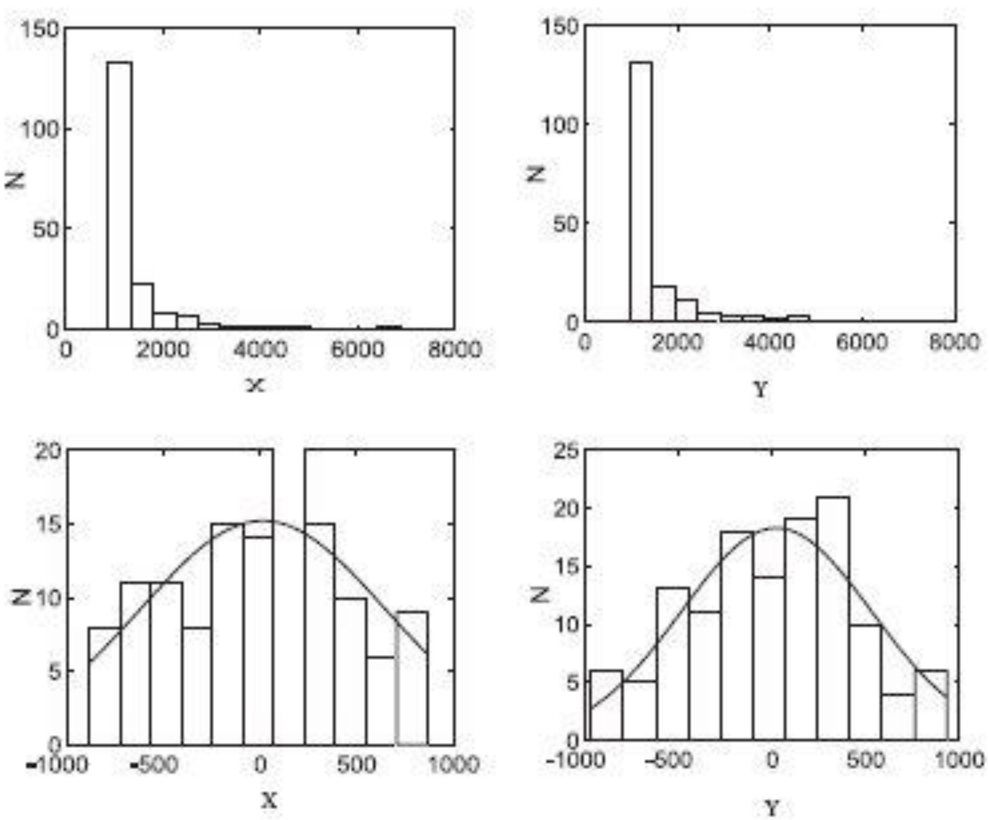
- Plot the histograms of the differences of the absolute distances
- The distributions are fitted with Gaussian functions

$$f(x) = \frac{A_x}{w_x \sqrt{\pi/2}} e^{-2(\frac{x-x_0}{\sigma x})^2}, \qquad f(y) = \frac{A_y}{w_y \sqrt{\pi/2}} e^{-2(\frac{y-y_0}{\sigma y})^2}, \qquad (6)$$

- The probability *p*, if a star is member of the star cluster is defined as

$$p_x = \mathrm{e}^{-2\left(\frac{x-x_0}{\sigma_x}\right)^2}, \qquad p_y = \mathrm{e}^{-2\left(\frac{y-y_0}{\sigma_y}\right)^2}. \tag{7}$$

$$p = p_x * p_y. \tag{8}$$



Javakhishvili et al., 2006, A&A, 447, 915 for Collinder 121

From these diagrams, the membership probability can be exactly determined

- In the same way, the proper motions in α and δ can be used, the basic equations and the determination of the membership probability is exactly the same

$$\delta\mu_{\alpha_i} = \sum_{j=1}^{N} |\mu_{\alpha_i} - \mu_{\alpha_j}|, \qquad \delta\mu_{\delta_i} = \sum_{j=1}^{N} |\mu_{\delta_i} - \mu_{\delta_j}| \tag{9}$$

$$\tilde{\delta}\mu_{\alpha_i} = \sum_{j=1}^{N} (\mu_{\alpha_i} - \mu_{\alpha_j}), \qquad \tilde{\delta}\mu_{\delta_i} = \sum_{j=1}^{N} (\mu_{\delta_i} - \mu_{\delta_j}). \tag{10}$$

But the errors of ground based proper motions are rather large, most catalogues are complete to V < 11 mag only. This limits us, currently, to distances of about 1000 pc.

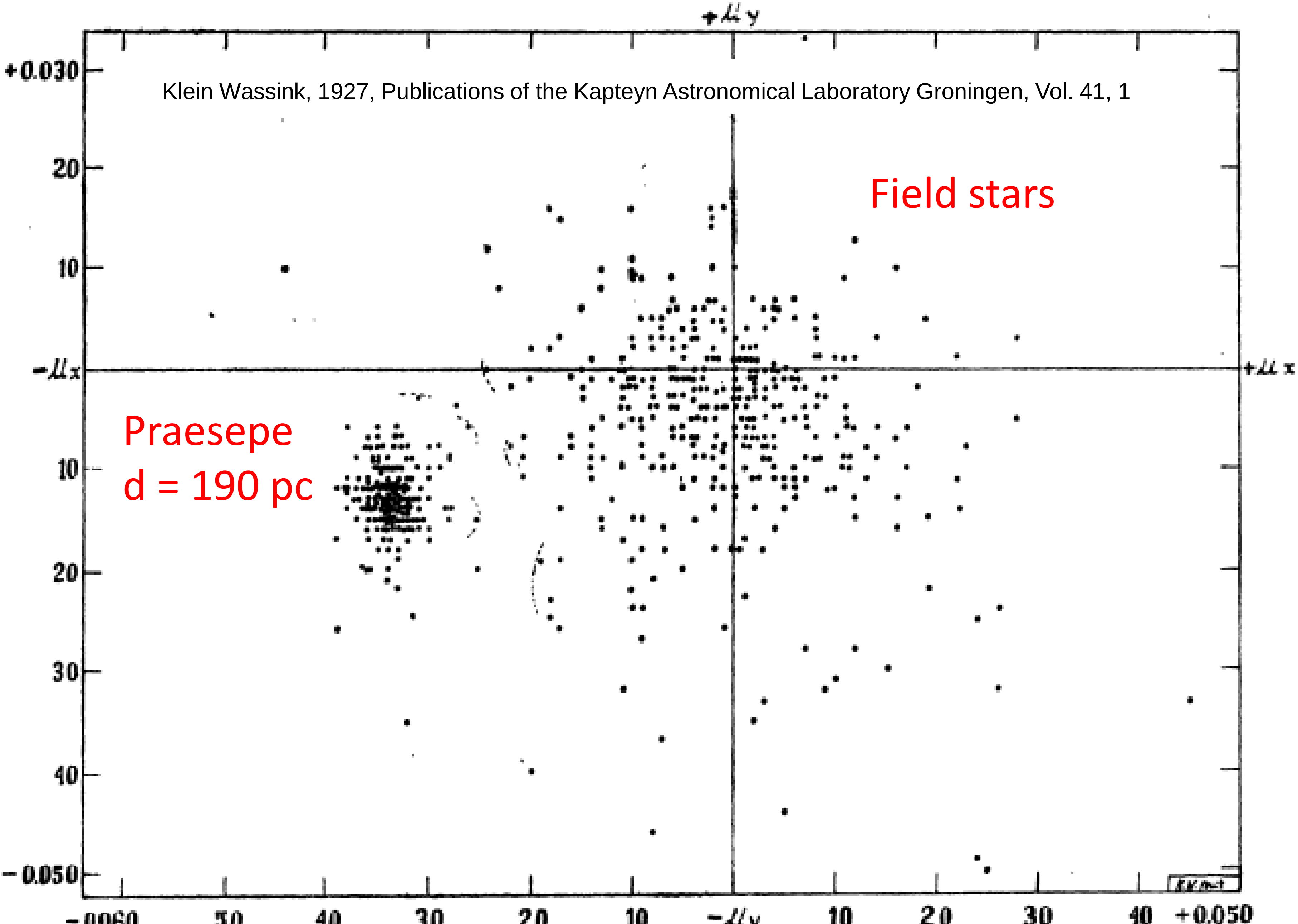
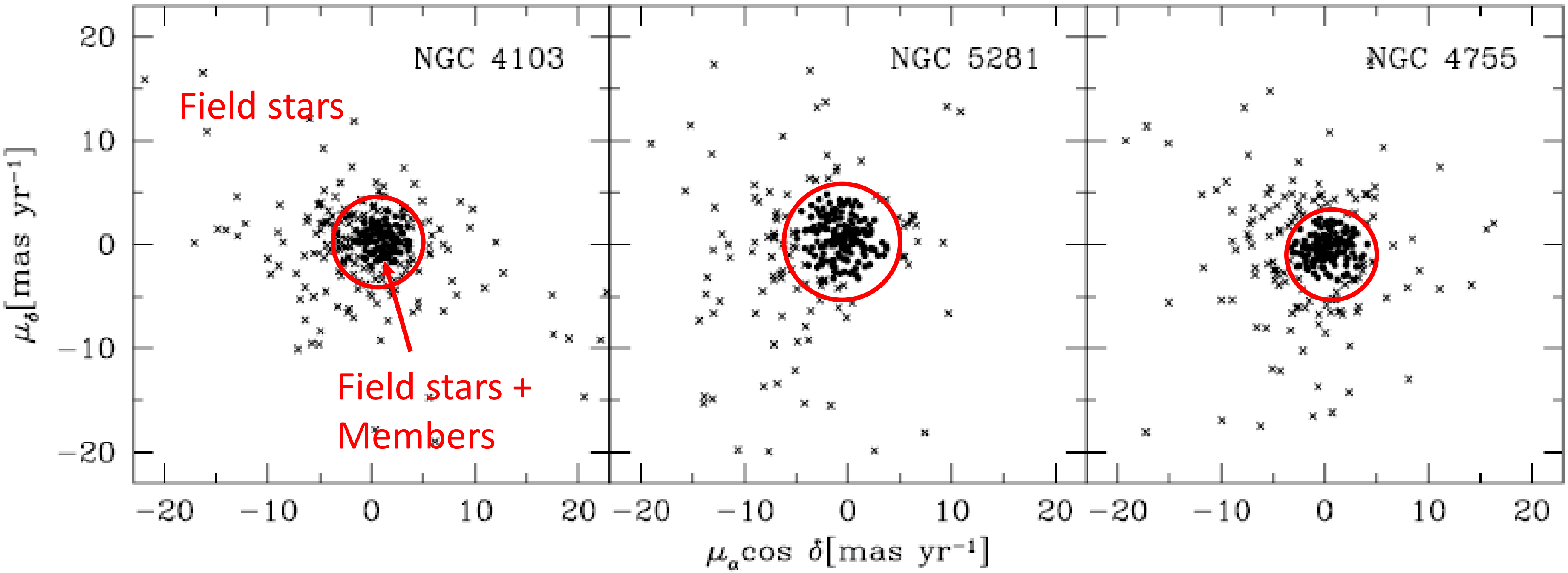


Figure 2. Diagram of the absolute proper motions of the Catalogue; photographic magnitude 6 to 14·0, numbers 1 to 531. The dotted lines separate the Praesepe stars from the backgroundstars.

Sanner et al., 2001, A&A, 369, 511

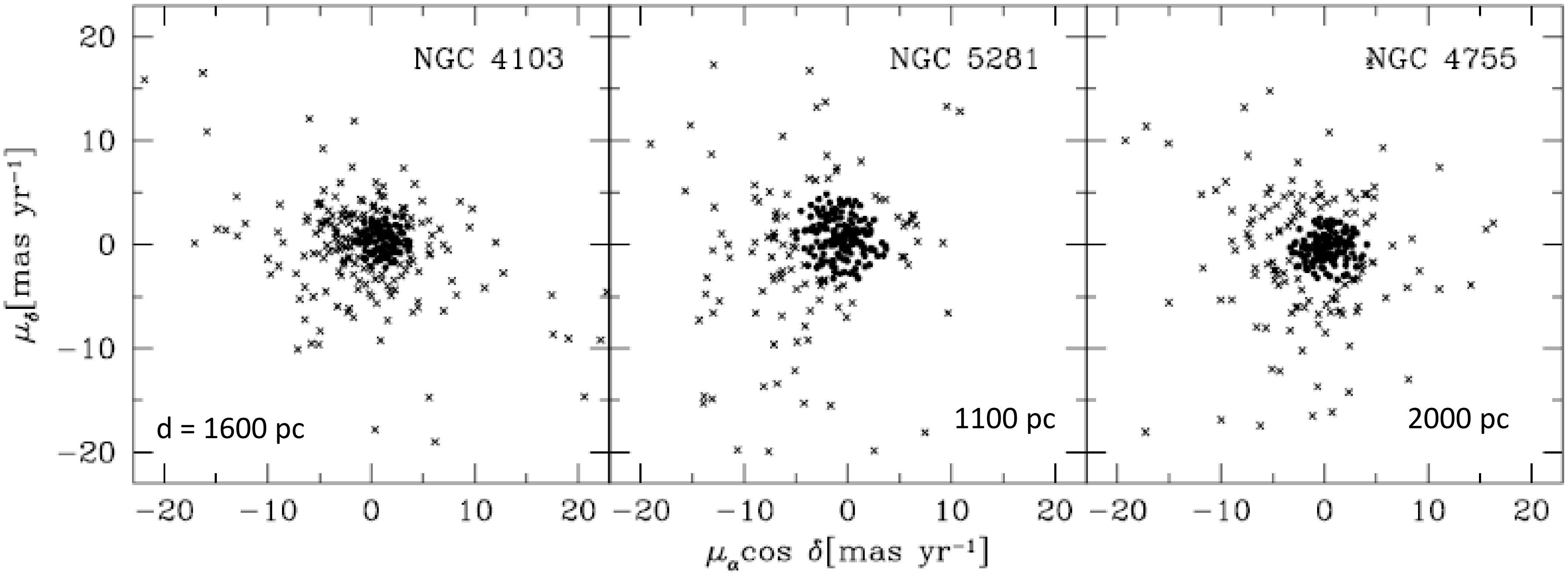


The proper motion for „distant" star clusters is almost zero.

Only field stars with large proper motions can be sorted out.

These are almost only foreground stars.

Sanner et al., 2001, A&A, 369, 511



object	$\mu_\alpha \cos\delta$ [mas yr^{-1}]	μ_δ [mas yr^{-1}]
NGC 4103	$+0.91 \pm 1.4$	$+0.36 \pm 1.4$
field	-0.92 ± 5.0	$+0.27 \pm 5.0$
NGC 5281	-0.70 ± 2.1	$+0.67 \pm 2.1$
field	-4.36 ± 7.0	-1.08 ± 7.0
NGC 4755	$+0.18 \pm 1.7$	-0.32 ± 1.7
field	-1.71 ± 6.5	-0.99 ± 6.5

Mean values

TYCHO2 data

$$\mu_\alpha \cos\delta = -6.4 \pm 4.6 \text{ mas yr}^{-1}$$
$$\mu_\delta = +0.3 \pm 3.9 \text{ mas yr}^{-1}$$
$$\mu_\alpha \cos\delta = -7.3 \pm 4.8 \text{ mas yr}^{-1}$$
$$\mu_\delta = -2.0 \pm 4.3 \text{ mas yr}^{-1}$$
$$\mu_\alpha \cos\delta = -2.9 \pm 3.9 \text{ mas yr}^{-1}$$
$$\mu_\delta = -1.3 \pm 4.3 \text{ mas yr}^{-1}$$

Absolute values after „Sun correction“

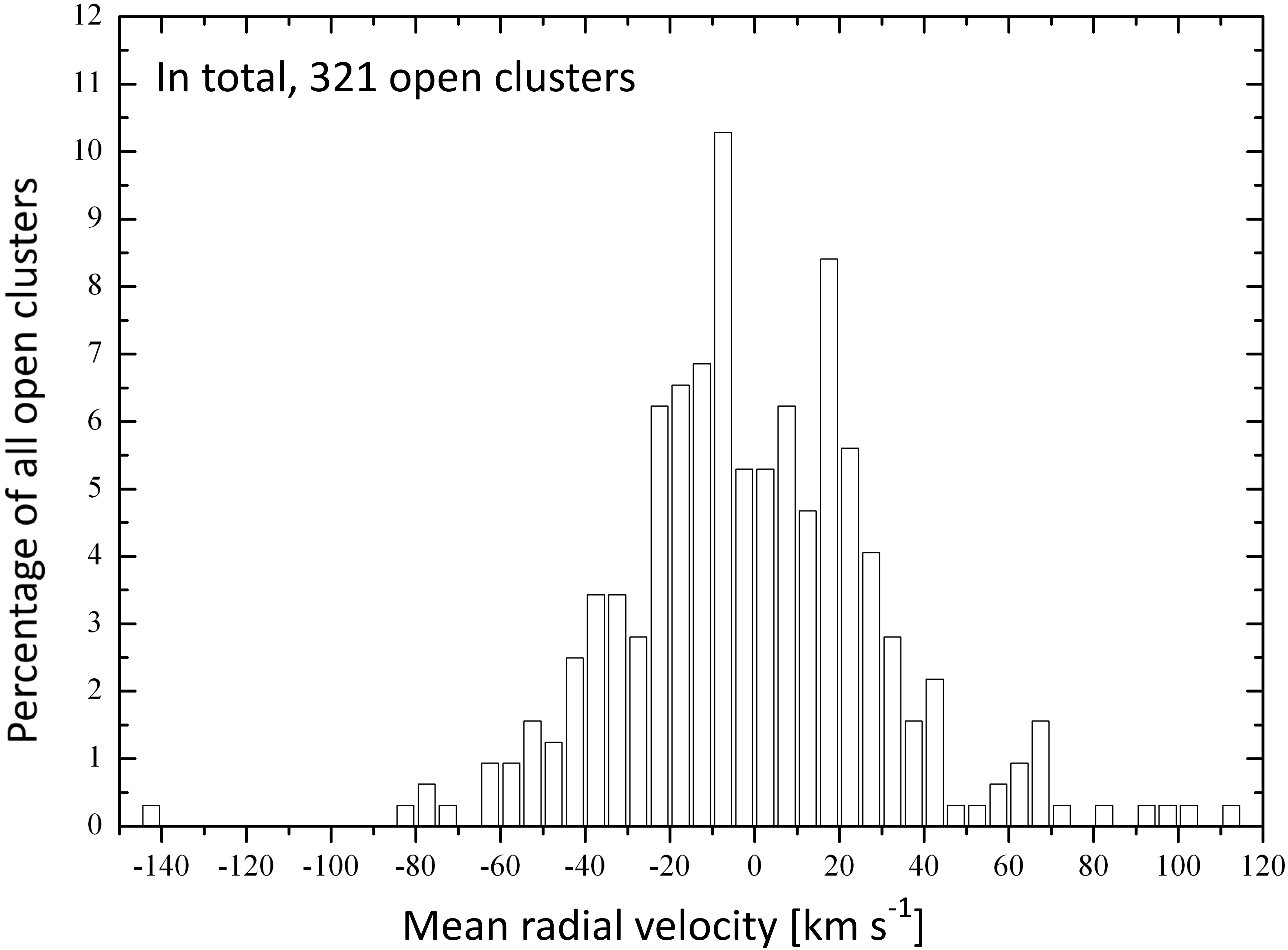
Radial velocities

- Advantages:

1. Correlated with the galactic rotation only
2. Possible to measure for most distant cluster members

- Disadvantages:

1. High-resolution high S/N spectrum needed
2. Faintness of members for distant clusters


In total, 321 open clusters
Percentage of all open clusters
Mean radial velocity [km s-1]
0
1
2
3
4
5
6
7
8
9
10
11
12
-140
-120
-100
-80
-60
-40
-20
0
20
40
60
80
100
120

Determination of the radial velocity

- Doppler shift of spectral lines

$$\Delta\lambda = \frac{v_R \lambda}{c}$$

- Determine the central wavelength of the shifted line
- Better accuracy if
  1. Instrumental resolution ($\lambda/\Delta\lambda$) is higher
  2. Signal-To-Noise ratio (S/N) is higher
  3. vsini of star is lower
  4. The number of measured lines is higher

λ [Å]	5	10	30	100	R_V [km s^{-1}]
3500	0,058	0,117	0,350	1,167	
4000	0,067	0,133	0,400	1,333	
4500	0,075	0,150	0,450	1,500	
5000	0,083	0,167	0,500	1,667	
5500	0,092	0,183	0,550	1,833	
6000	0,100	0,200	0,600	2,000	
6500	0,108	0,217	0,650	2,167	
7000	0,117	0,233	0,700	2,333	
7500	0,125	0,250	0,750	2,500	
8000	0,133	0,267	0,800	2,667	$\Delta\lambda$ [Å]

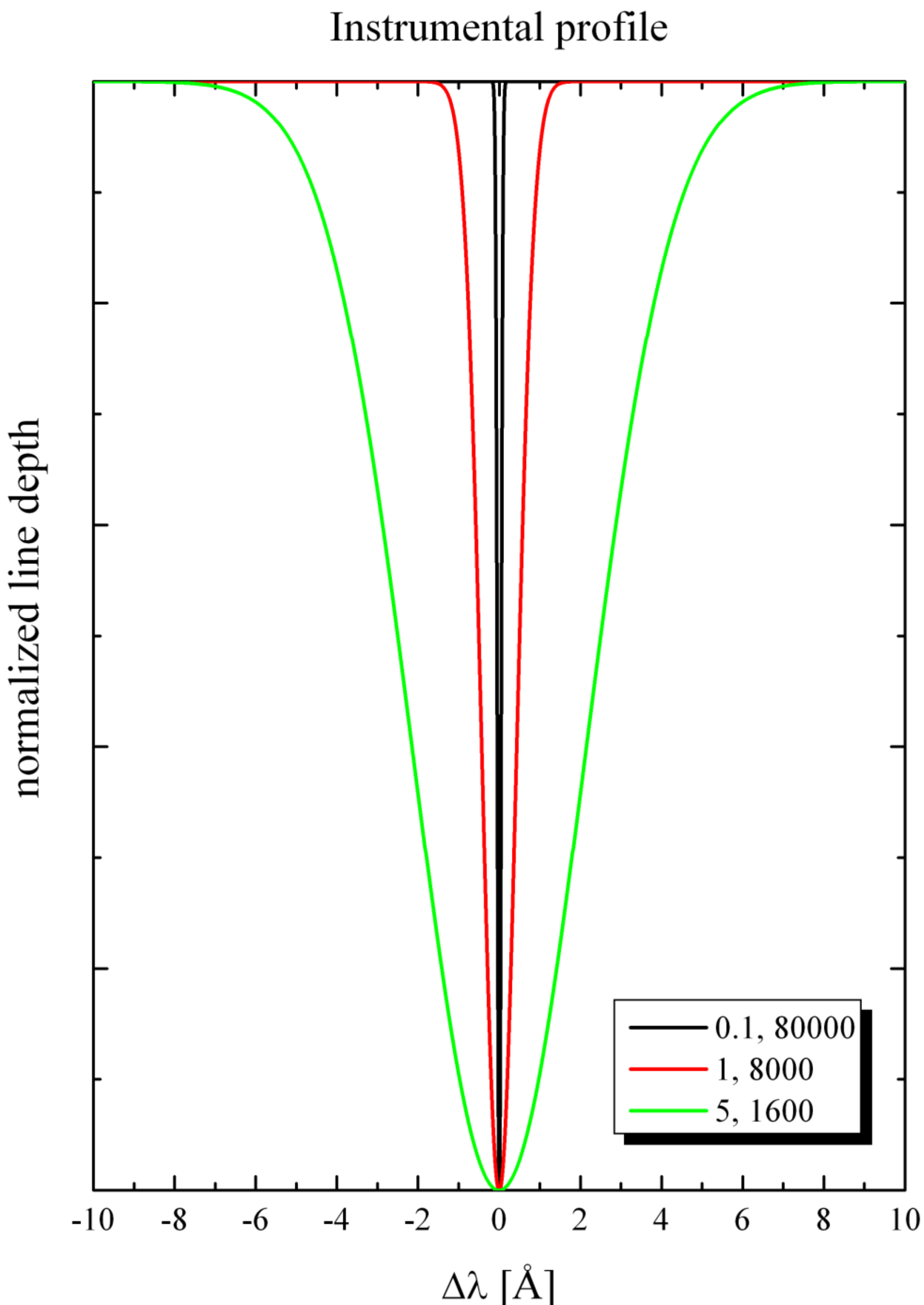
Instrumental profile defined by the resolution:

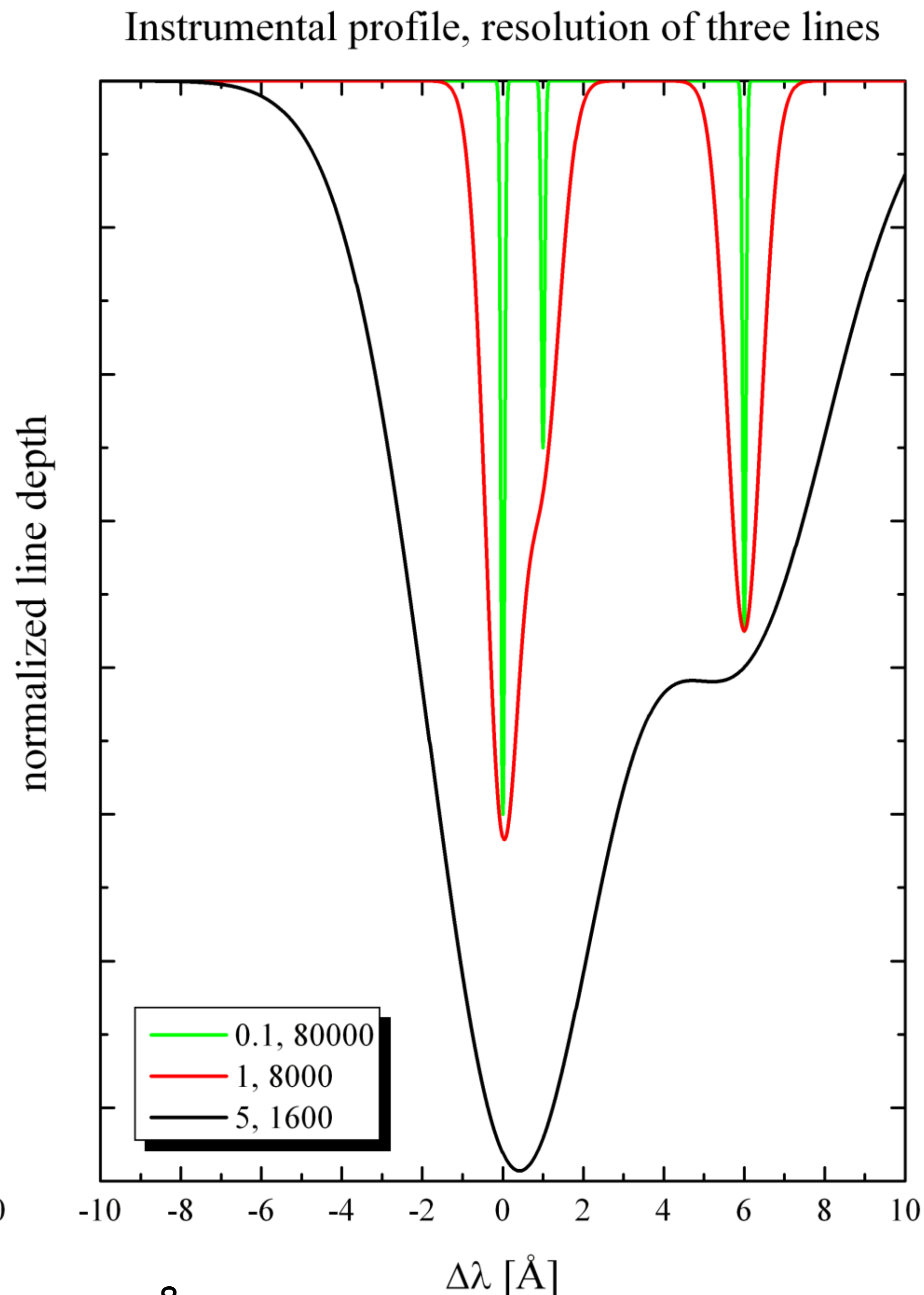
$$IP(\Delta\lambda) = \exp\left[-0.5\left(\frac{(\lambda - \Delta\lambda)}{\sigma}\right)^2\right] \text{ with } \sigma = \frac{FWHM}{2.355}$$

Rotational broadening:

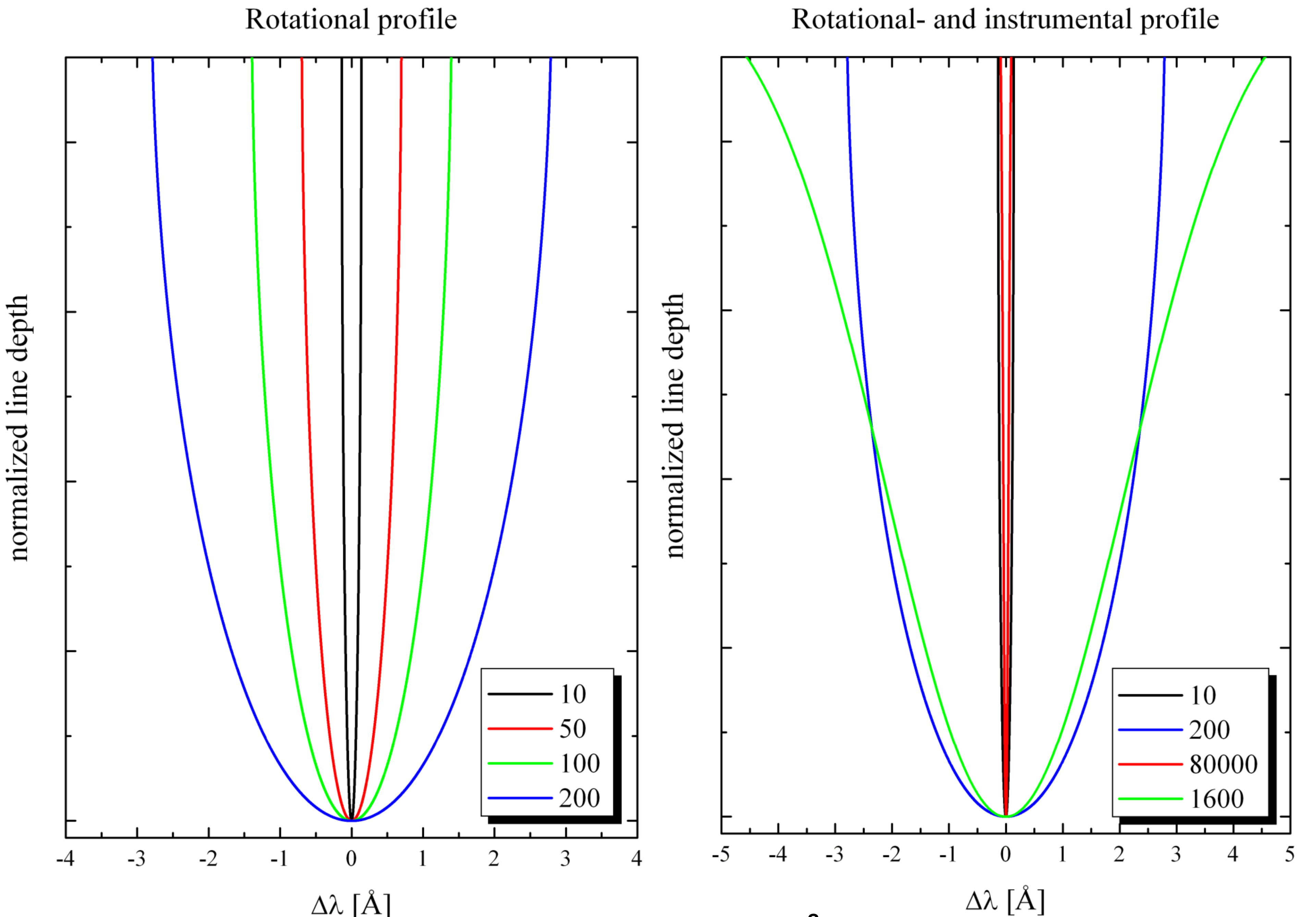
$$RP(\Delta\lambda) = c_1\sqrt{x} + c_2 x \quad \text{with} \quad x = 1 - \left(\frac{\Delta\lambda}{\Delta\lambda_L}\right)^2$$

$$\Delta\lambda_L = \lambda \frac{v \sin i}{c}$$





for λ = 4200Å

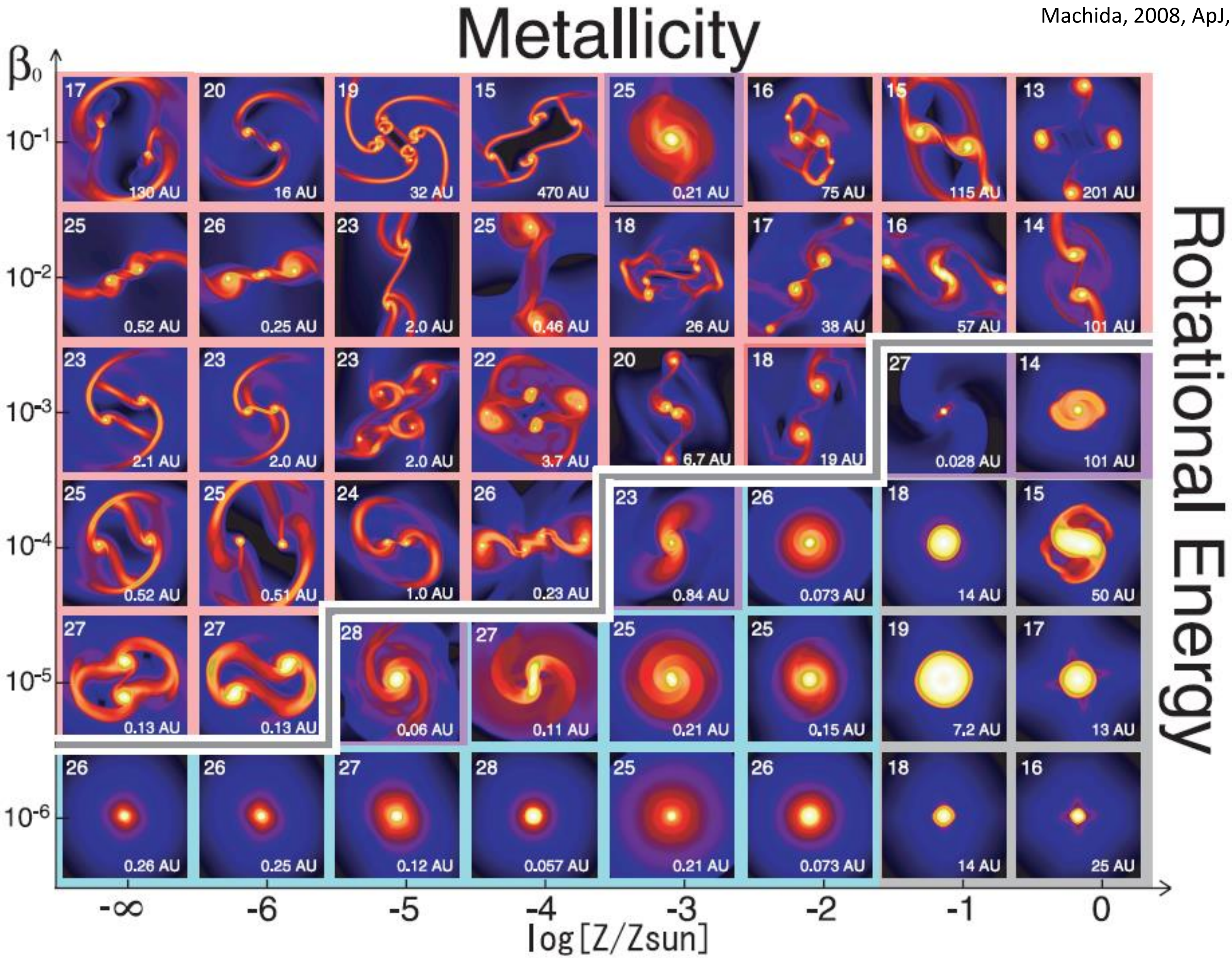


for λ = 4200Å

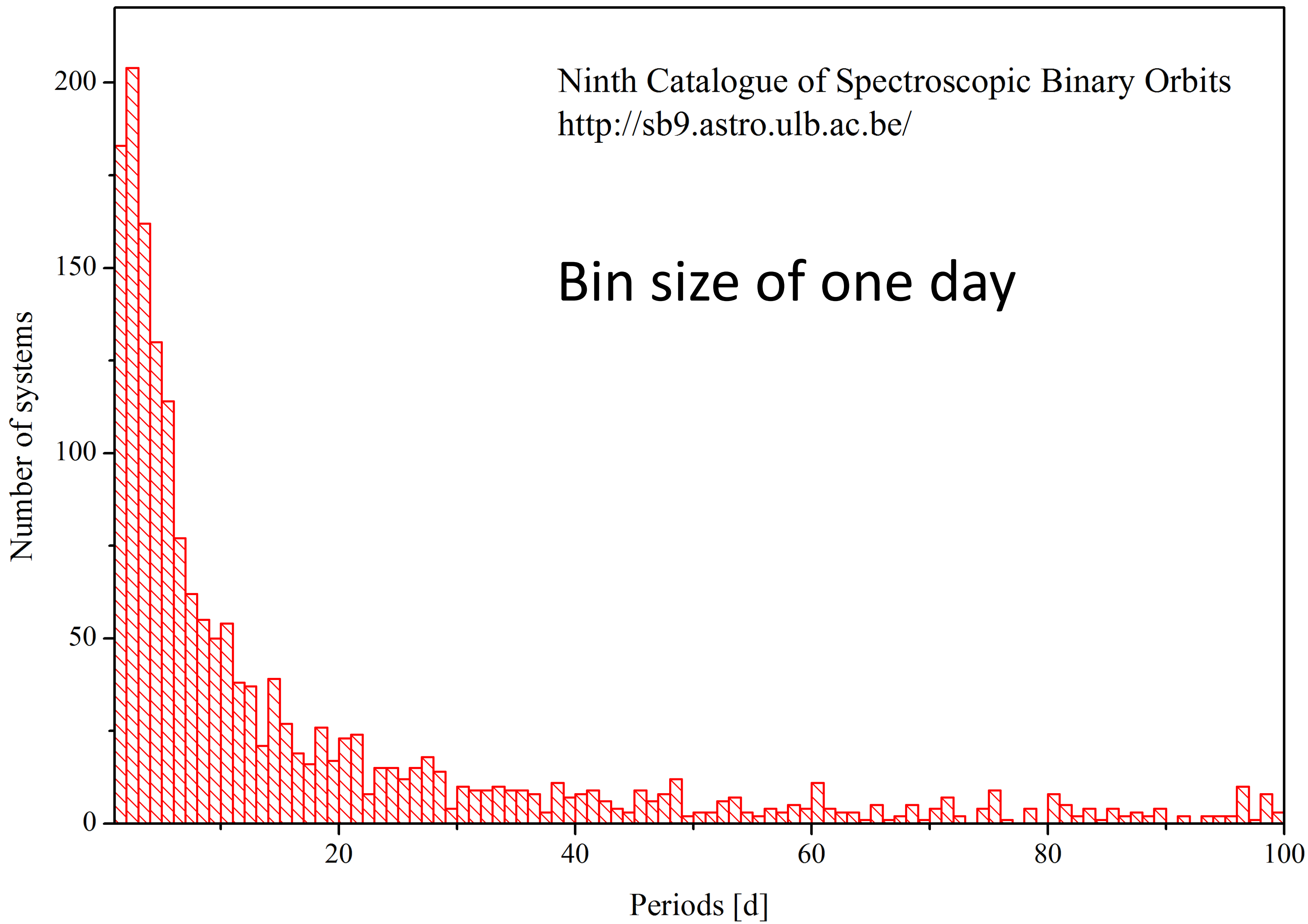
Binary fraction

- Important for the formation and evolution of star clusters
- Critical parameter for the IMF
- Needed for N-body numerical simulations
- Observations are biased in many respects
- Many different types of binary systems

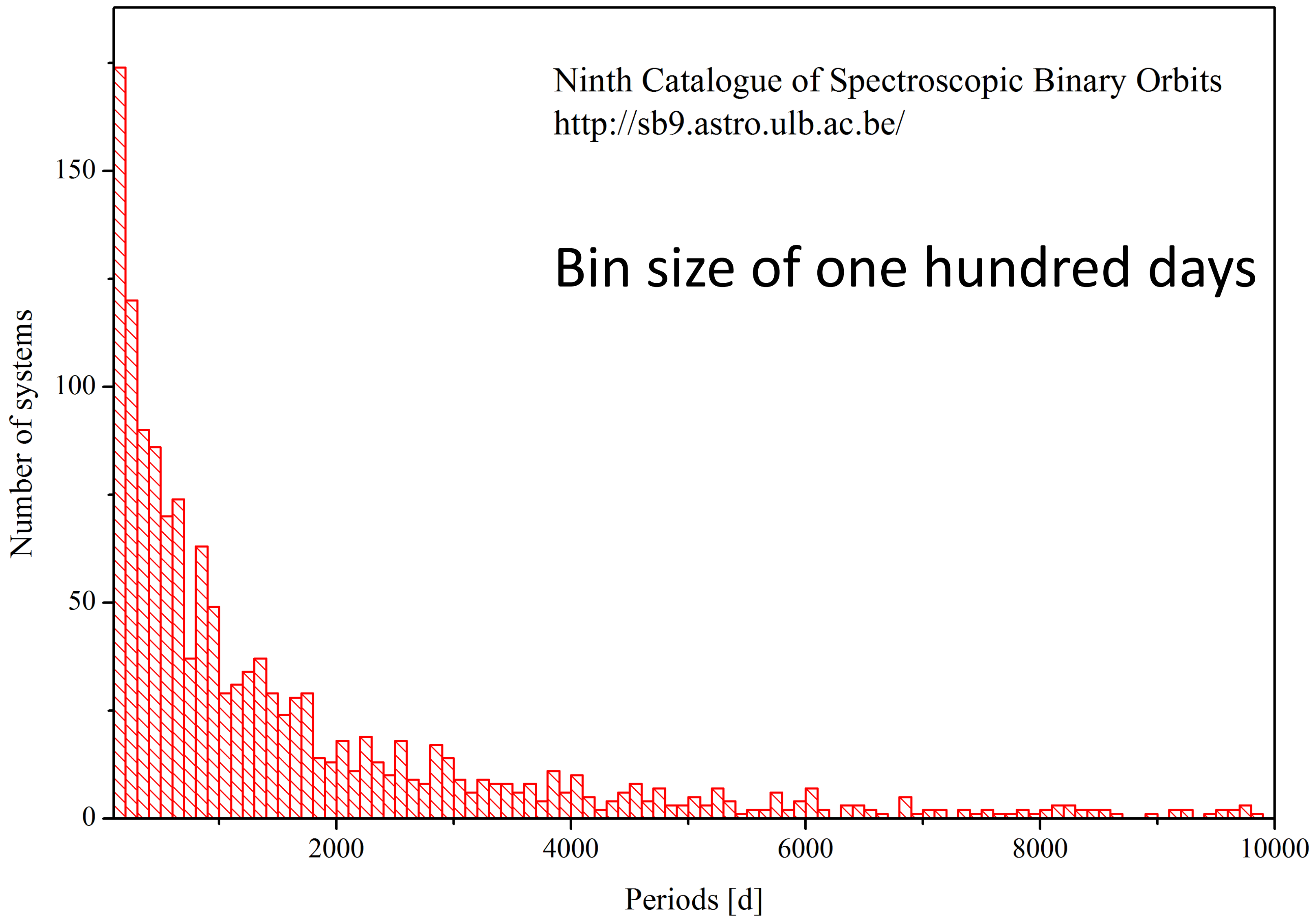
Machida, 2008, ApJ, 682, L1



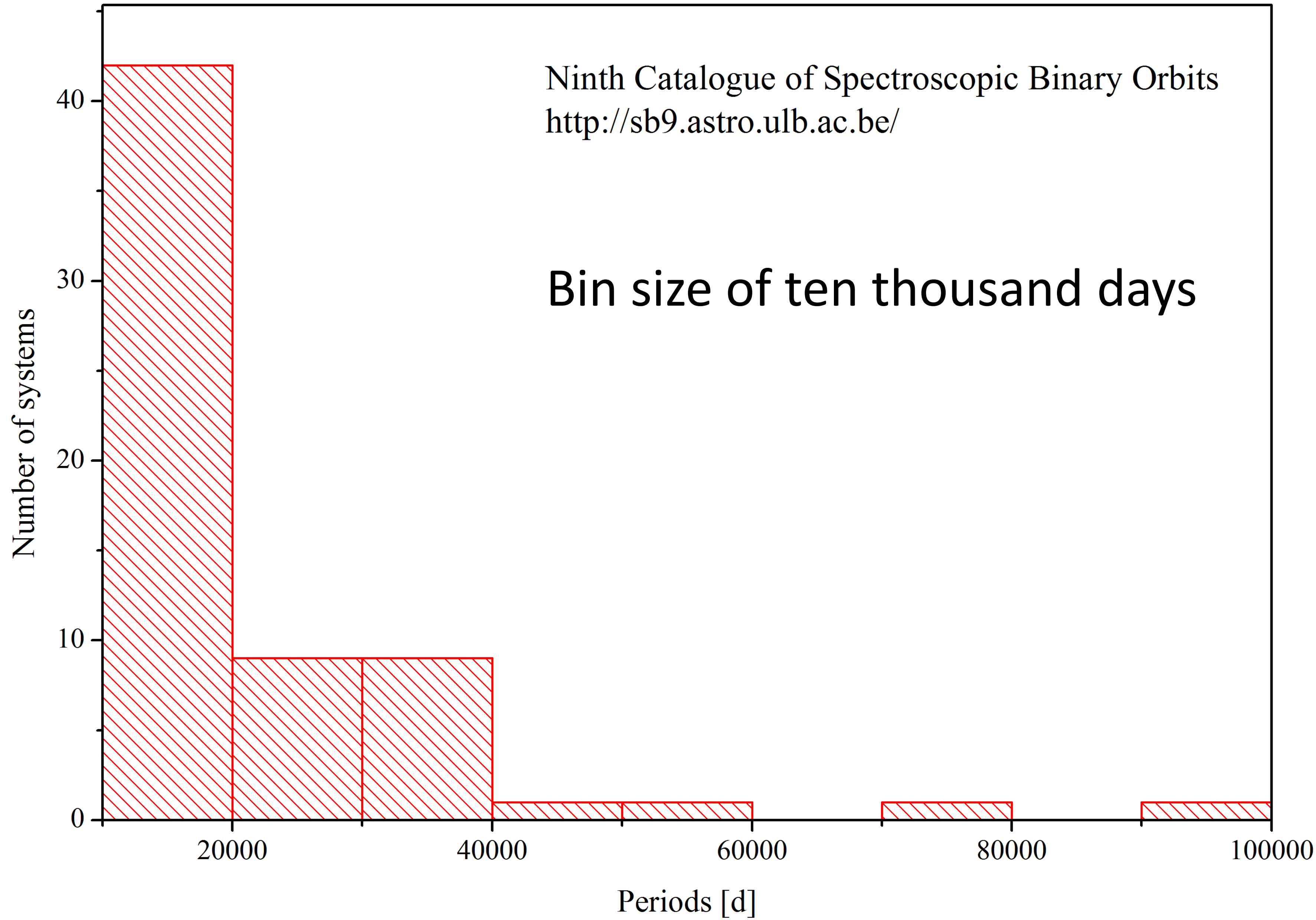
Lower metallicities seem to favour binary formation



Ninth Catalogue of Spectroscopic Binary Orbits
http://sb9.astro.ulb.ac.be/
Bin size of one day
Number of systems
200
150
100
50
0
20
40
60
80
100
Periods [d]



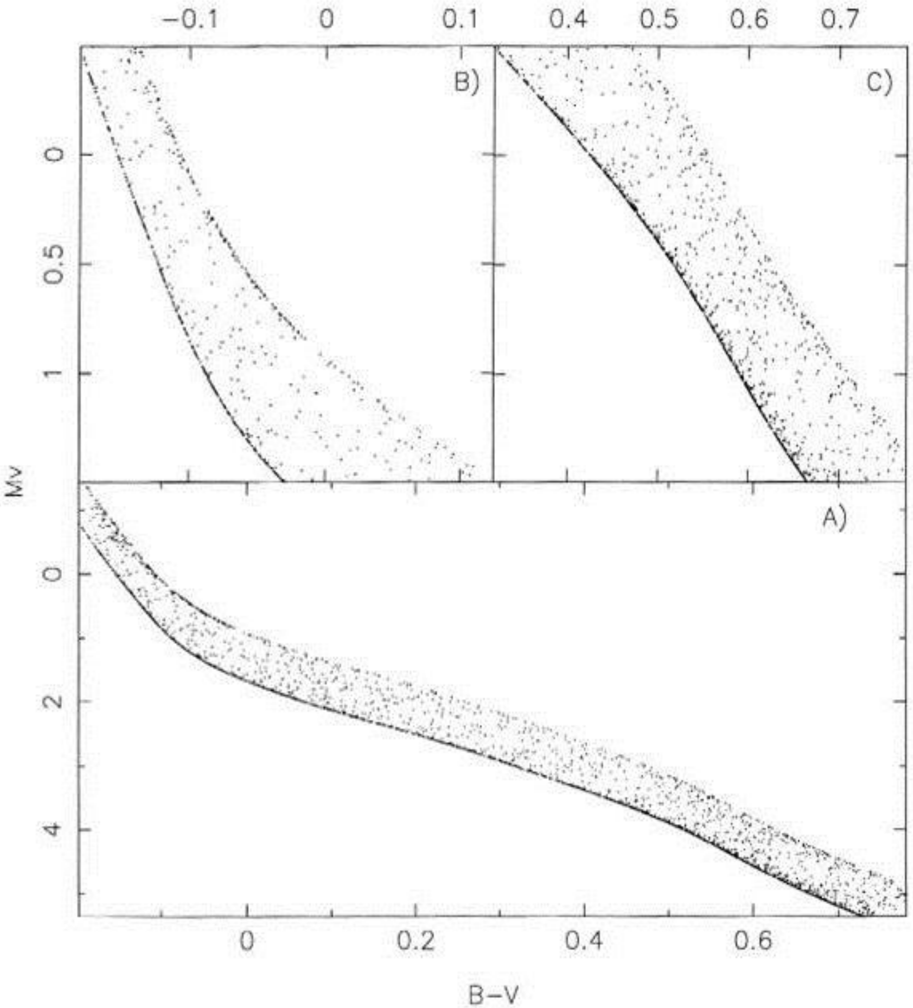
Ninth Catalogue of Spectroscopic Binary Orbits
http://sb9.astro.ulb.ac.be/
Bin size of one hundred days
Number of systems
0
50
100
150
2000
4000
6000
8000
10000
Periods [d]


Ninth Catalogue of Spectroscopic Binary Orbits
http://sb9.astro.ulb.ac.be/
Bin size of ten thousand days
Number of systems
0
10
20
30
40
20000
40000
60000
80000
100000
Periods [d]

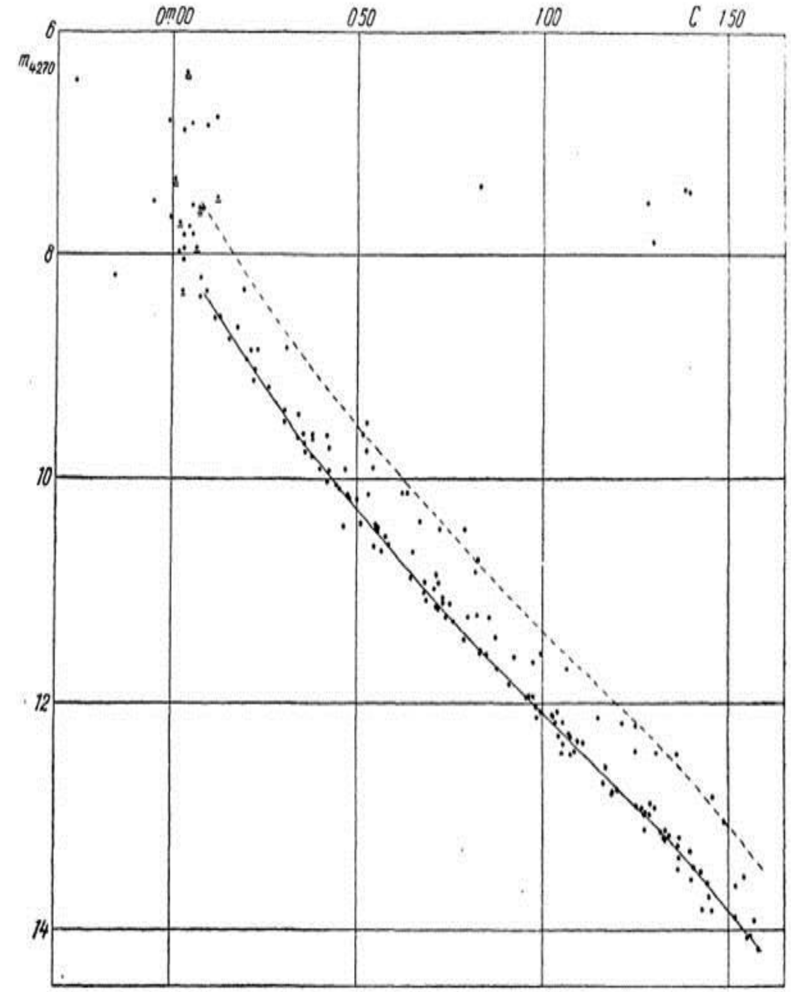
How to observe the binary fraction?

- Photometric observations of star clusters
  1. “Cluster main sequence”
  2. Eclipsing binaries
  3. Positions (astrometric binaries)
- Spectroscopic observations
  1. Radial velocity variability
  2. Direct detection in spectrum (SB2)

Hurley & Tout, 1998, MNRAS, 300, 977



Haffner & Heckmann, 1937, VeGeo, 55, 77



Simulation with randomly distributed mass ratios

Observations of Praesepe with known binary systems

Results for open clusters

- Sollima et al., 2010, MNRAS, 401, 577
  - NGC 188 (9.63): 21 – 58%
  - NGC 2204 (9.20): 12 – 36%
  - NGC 2243 (9.58): 34 – 70%
  - NGC 2420 (9.08): 17 – 51%
  - NGC 2516 (8.52): 25 – 66%
- Sana et al., 2009, MNRAS, 400, 1479
  - NGC 6611 (6.50): 44 – 67%
- Sana et al., 2008, MNRAS, 386, 447
  - NGC 6231 (6.50): 63% - ?
- Bica & Bonatto, 2005, A&A, 431, 943
  - IC 4651 (9.26): 50 +- 11%
  - NGC 2287 (8.20): 48 +- 45%
  - NGC 2447 (8.60): 21 +- 9%
  - NGC 2548 (8.56): 48 +- 23%
  - NGC 2682 (9.51): 39 +- 16%
  - NGC 3680 (9.20): 25 +- 5%
  - NGC 5822 (9.00): 16 +- 8%
  - NGC 6208 (9.11): 54 +- 30%
  - NGC 6694 (7.85): 18 +- 12%
- Sandhu et al., 2003, A&A, 408, 515
  - NGC 2099 (8.60): ~30%
  - King 5 (9.00): ~30%
  - King 7 (8.80): ~20%

Globular clusters

	f
NGC 288	0.15 ± 0.05
	>0.06
NGC 362	0.21 ± 0.06
NGC 2808	
NGC 3201	
NGC 4590	>0.09
NGC 5053	>0.08
NGC 5466	>0.08
NGC 5897	>0.07
NGC 6101	>0.09
NGC 6362	>0.06
NGC 6397	<0.07
NGC 6723	>0.06
NGC 6752	0.27 ± 0.12
NGC 6792	
NGC 6981	>0.10

M3	
	0.14 ± 0.08
M4	$0.23^{+0.34}_{-0.23}$
M15	~ 0.07
M22	
M30	
M55	>0.06
M71	$0.22^{+0.26}_{-0.12}$
M92	
Arp 2	>0.08
Terzan 7	>0.21
Palmoar 12	>0.18
Palmoar 13	0.30 ± 0.04
47 Tucane	0.14 ± 0.04

Davis et al., 2008, AJ, 135, 2155