

Tidal disruption events (TDEs): properties & models



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Astronomical transients

Selected chapters from astrophysics, fall semester, 2022

TDE phenomenon

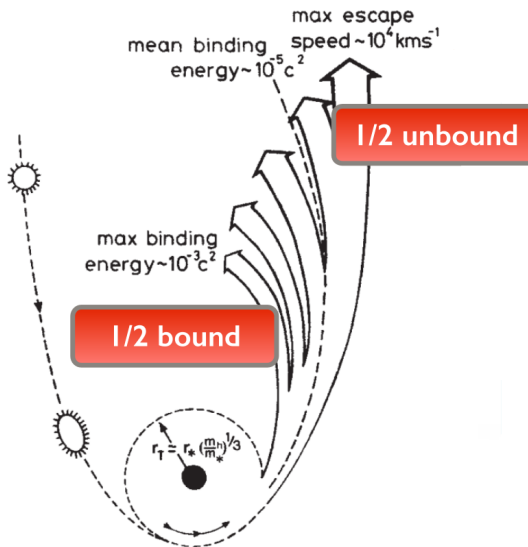
cf. T. Piran's &

E. M. Rossi's talk on 35HUJI

Credit: Stephan Rosswog 2006

TDE phenomenon

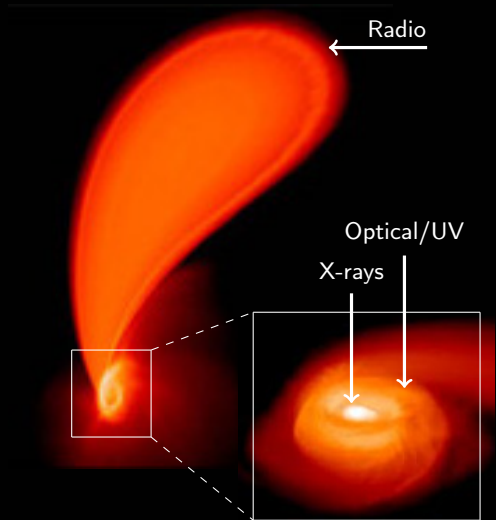
- Tidal disruption occurs when the tidal gravitational field has comparable magnitude to the self-gravity of the star, occurring at a radius given by
$$R_t \approx R_* \left(\frac{M_{\text{BH}}}{M_*} \right)^{1/3}$$
- Gas particles are then ejected on nearly ballistic orbits
- Disrupted stars are typically on approximately parabolic orbits
- Half of the material is bound while the other half forms an unbound collimated stream



Credit: Martin Rees 1988

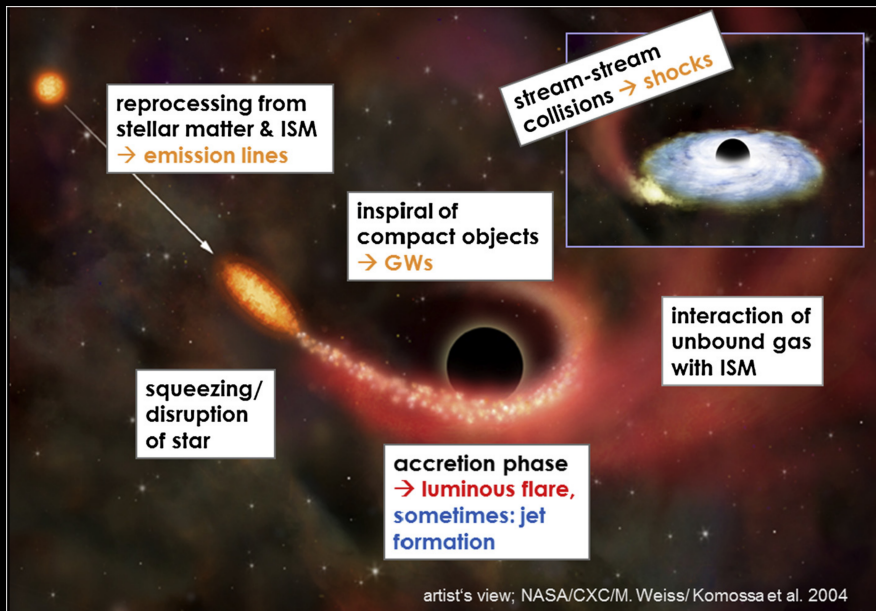
TDE phenomenon

- Radio \rightarrow interaction zone of debris with CSM \rightarrow very similar to GRB afterglows (see previous lecture)
- Differences \rightarrow not relativistic & very far from spherical \rightarrow the role of shock waves basically similar
- Optical/UV comes from the central blob of matter \rightarrow from the distance $\sim 10^{15}$ cm out of the center; from the region where the two streams interact
- The X-rays come from the very tiny region of the accretion process around the center



Credit: Rosswog+ 2009

TDE phenomenon



TDE process

- Tidal radius:

$$R_t \sim R_\star \left(\frac{M_{\text{BH}}}{M_\star} \right)^{1/3} \approx 30 \text{ (will derive later)}$$

- Energy E_t necessary to bring the star to R_t from “infinity”:

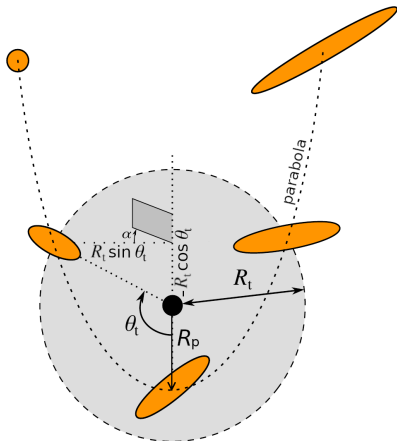
$$E_t \approx \frac{GM_{\text{BH}}M_\star}{R_\star} \sim \frac{M_\star c^2}{R_t/R_{\text{schw}}} \approx 10^{52} \text{ ergs}$$

- Hydrodynamic timescale τ_h to disrupt the star:

$$\tau_h \sim \left(\frac{GM_\star}{R_\star^3} \right)^{-1/2} \approx 10^3 - 10^4 \text{ s}$$

- The specific energy ΔE necessary to disrupt the star:

$$\Delta E \sim \pm \frac{GM_{\text{BH}}}{R_t^2} R_\star \text{ (will derive later)} \sim 10^{20} \text{ erg} \sim \frac{v_\infty^2}{2} \Rightarrow v_\infty \sim 10^4 \text{ km s}^{-1}$$



Credit: Bonnerot+ 2017

Dynamics & rates \rightarrow TDEs

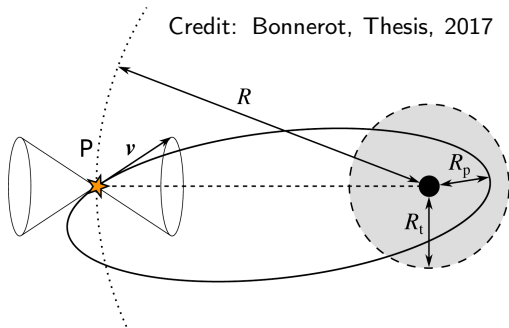
cf. Re'em Sari's talk on 35HUJI

- **Dynamic processes** near a galactic center leading to TDEs
- Rates: **Loss cone** \Rightarrow full/empty
- **Relaxation timescale** \rightarrow 2-body (resonance relaxation)
- Evaluating the ratio $\frac{\mathcal{R}_{\text{TDE}}}{\mathcal{R}_{\text{EMRI}}}$ (Extreme-Mass-Ratio-Inspiral: stars orbiting around BH emitting GWs \rightarrow get closer to the BH via slowly inspiralling on \sim circular orbits)
- Distribution of stars around BH: **Bahcall-Wolf (BW) cusp** $\rightarrow \rho \sim r^{-7/4} \rightarrow$ may affect the loss cone rates
- **Galactic central BH mass:** $M_{\text{BH}} = 4 \times 10^6 M_{\odot} \Rightarrow R_{\text{Schw}} = \frac{2GM_{\text{BH}}}{c^2} \approx 0.1 \text{ AU}$
- **Radius of influence R_{inf} :** radius of a sphere where the mass of stars equals the $M_{\text{BH}} \Rightarrow M_{\text{tot}}(r < R_{\text{inf}}) = 2M_{\text{BH}}$; $R_{\text{inf}} \sim 2 \text{ pc}$ in our galaxy $\Rightarrow \frac{R_{\text{inf}}}{R_{\text{Schw}}} \approx 4 \times 10^6$

Loss cone: full/empty

Credit: Bonnerot, Thesis, 2017

- We now ignore the fact that stars have different masses \Rightarrow
 $\langle M_\star \rangle \cong 1 M_\odot \rightarrow$ # of stars within the $R_{\text{inf}} \cong 4 \times 10^6$
- Circular Keplerian orbital period P at $R_{\text{inf}} \approx 10^5$ yrs



- Tidal (residual) differential acceleration exerted by BH on a star with mass $M_\star = M_\odot$ and radius $R_\star = R_\odot$, at a distance $r \gg R_\star$:

$$a_t = \frac{GM_{\text{BH}}}{r^2} - \frac{GM_{\text{BH}}}{(r + R_\star)^2} \approx \frac{2GM_{\text{BH}}R_\star}{r^3}$$

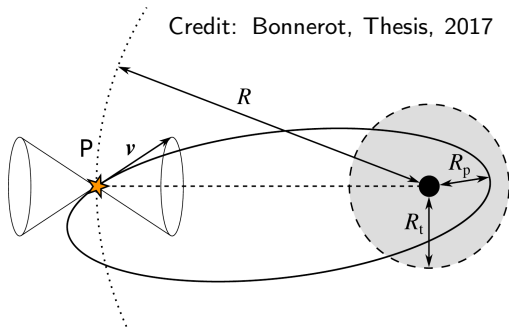
- We expect that if $a_t \geq g_\star$, the star will be tidally disrupted: we define the tidal radius $r = R_t$ where $a_t = g_\star \equiv GM_\star/R_\star^2$,

$$R_t \approx R_\star \left(\frac{2M_{\text{BH}}}{M_\star} \right)^{1/3} \approx 1 \text{ AU} \Rightarrow \frac{R_t}{R_{\text{schw}}} \approx 10 \quad (\text{in our galaxy})$$

Loss cone: full/empty

Credit: Bonnerot, Thesis, 2017

- Let's assume all 4×10^6 stars concentrated in or at R_{inf} with isotropic distribution of velocity \mathbf{v} : how many stars will have the pericenter distance R_p of their elliptical orbits below R_t ?



- Conservation of angular momentum: $j = V_{\text{orb}} R_{\text{inf}} = \sqrt{GM_{\text{BH}} R_{\text{inf}}}$ for a star orbiting at R_{inf} with the Keplerian orbital velocity, per unit mass
- Angular momentum on an Keplerian orbit scales as $\sim \sqrt{r} \Rightarrow$ only stars in the point P with j lower by $\sqrt{R_t/R_{\text{inf}}}$ compared to j_{circ} can get below $R_t \Rightarrow$ only a fraction of orbits $f_{\text{orb}} \left(j < \sqrt{\frac{R_t}{R_{\text{inf}}}} j_{\text{circ}} \right) = \frac{R_t}{R_{\text{inf}}}$ (2D cone section!)
- The latter ratio $= (4 \times 10^{-5})^{-1} \Rightarrow$ from the $\#$ of isotropically moving stars in the point P, only 10 may have sufficiently small j

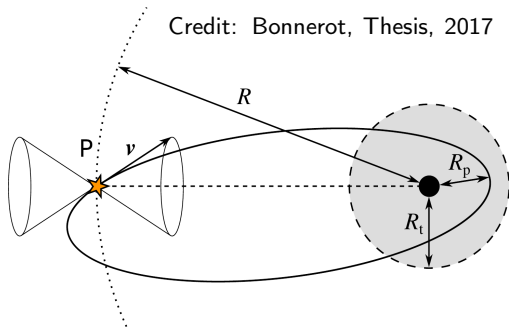
Loss cone: full/empty

Credit: Bonnerot, Thesis, 2017

- This means: every 10^5 years (P_{orb} on R_{inf}), we will have 10 TDEs \Rightarrow **full loss cone**:

$$\mathcal{R}_{\text{TDE}} = \frac{N_{\star} \frac{R_t}{R_{\text{inf}}}}{P} = \frac{M_{\text{BH}}}{M_{\star}} \frac{R_t}{R_{\text{inf}}} \frac{1}{P}$$

- $\mathcal{R}_{\text{TDE}} \sim 10/P \rightarrow$ but: only in case of **no interactions** between orbits!



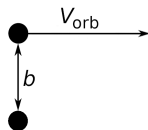
- After some time will all stars within this cone in velocity space be **tidally destroyed**: we obtain the **empty loss cone** $\Rightarrow \mathcal{R}_{\text{TDE}} = 0$
- **Next step**: how quickly the stars scatter each other to **refill the loss cone** that the BH evacuated?

Loss cone: full/empty

- 2 stars moving **absolutely randomly** at or near R_{inf} : one passes the other with **velocity** v_{orb} , with an **impact parameter** b ; the **change in j** is given by mutual gravity times the timescale of the interaction b/v_{orb} , times R_{inf} :

$$\Delta j = \frac{GM_{\star}}{b^2} \frac{b}{v_{\text{orb}}} R_{\text{inf}} = \frac{GM_{\star}}{b v_{\text{orb}}} R_{\text{inf}}$$

$gt = v$



- What is the **rate of interactions at a distance b** ? A cylinder with a volume $V = 1/\rho \rightarrow$ a cross-section $S = \pi b^2$ and a length Δx :

$$\frac{1}{t} = \frac{\rho S \Delta x}{t} \approx \pi b^2 \frac{N_{\star}}{R_{\text{inf}}^3} v_{\text{orb}} = \pi b^2 \frac{M_{\text{BH}}}{M_{\star}} \frac{1}{R_{\text{inf}}^3} v_{\text{orb}}$$

- What will be the value of $b \rightarrow$ **to deflect an orbit by 90°** ? Change in the “red” velocity gt will be v_{orb} (in case of 180° deflection $\rightarrow 2v_{\text{orb}}$ - see the 5th lecture):

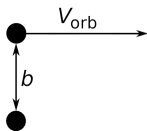
$$b_{\text{min}}: \frac{GM_{\star}}{b v_{\text{orb}}} = v_{\text{orb}} \Rightarrow b_{\text{min}} = \frac{GM_{\star}}{v_{\text{orb}}^2} \equiv \frac{M_{\star}}{M_{\text{BH}}} R_{\text{inf}}$$

- So, **over the timescale** for which these interactions occur, the velocity distribution will be completely **reconfigured** \rightarrow “**isotropised**”

Loss cone: full/empty

- What will be the **timescale** of these interactions? The **inverse of the rate** of interactions with b_{\min} will be the **relaxation time** (+ a small logarithmic factor added later):

$$T_{\text{relax}} \sim \left[b^2 \frac{M_{\text{BH}}}{M_{\star}} \frac{1}{R_{\text{inf}}^3} v_{\text{orb}} \right]^{-1} = \frac{M_{\text{BH}}}{M_{\star}} P \quad (\sim 10^{11} \text{ yrs}?)$$



- We have to **correct this time** by a logarithmic factor that comes from the fact that **most of interactions are weaker** even if more frequent:

$$T_{\text{relax}} \sim \frac{M_{\text{BH}}}{M_{\star}} P \frac{1}{\ln \frac{M_{\text{BH}}}{M_{\star}}} \quad (\sim 10^{10} \text{ yrs} \approx \text{age of Galaxy})$$

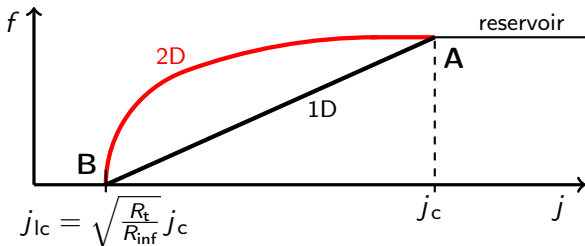
- But: for our purpose, I do not need to isotropise the j of **all stars** but only of a tiny **fraction of them** that is necessary to **refill the loss cone**:

$$T_{\text{relax,LC}} = \frac{R_t}{R_{\text{inf}}} T_{\text{relax}} \approx P$$

- This holds **if the edges of the loss cone are sharp** \rightarrow they are diluted so you have to “bring” stars from farther away $\rightarrow T_{\text{relax}} \approx P \times \ln \frac{M_{\text{BH}}}{M_{\star}} \approx 10 P$

Alternative approach - j diffusion

- I can regard the previous also as a **diffusion flux in 2D** in a j space \rightarrow pointing out that the **1st Fick's law in 2D** has a specific property



- We try to diffuse a quantity $f(j)$ from the point A where it has a constant distribution to the point B where it is cleaned
- Why in 2D: **only 2** components of v **contribute to j** (the 3rd - radial - does not)
- The 1st Fick's law in 1D: $D \frac{\partial f}{\partial j} = \text{const.}$ \rightarrow **the flux is linear** (black line)
- The 2D 1st Fick's law: $j D \frac{\partial f}{\partial j} = \text{const.}$ $\Rightarrow f \propto \ln \frac{j}{j_{lc}}$ \rightarrow particles (stars) have to diffuse through inwardly smaller ring areas that becomes "harder" (red line)

BW cusp

- What will be the **rate** of 2B interactions in case of **any given distance** r :

$$\begin{aligned}\frac{1}{t} &\sim b_{\min}^2 \frac{N_*(r)}{r^3} v(r) = \frac{G^2 M_*^2 N_*(r)}{v(r)^3 r^3} = \frac{G^2 M_{\text{BH}}^2 N_*(r)}{v(r)^3 r^3} \left(\frac{M_*}{M_{\text{BH}}} \right)^2 = \\ &= N_*(r) \frac{G M_{\text{BH}}}{r^3} \frac{r}{v(r)} \left(\frac{M_*}{M_{\text{BH}}} \right)^2 = N_*(r) \left(\frac{M_*}{M_{\text{BH}}} \right)^2 \frac{1}{P(r)} \quad (+ \text{ the log factor...})\end{aligned}$$

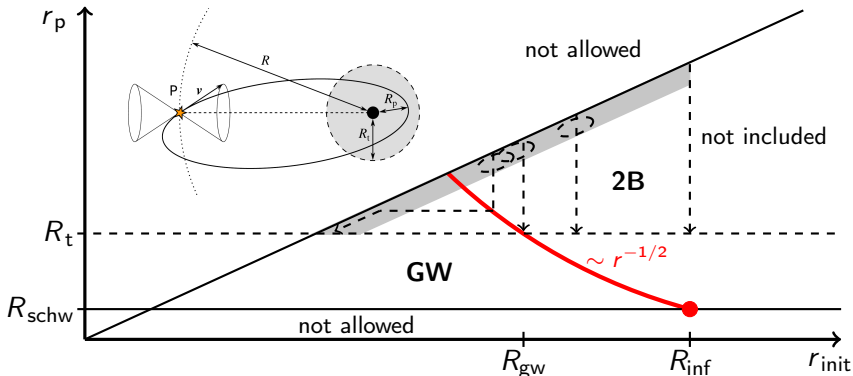
- **Peebles**: BH absorbs stars that arrive as a **constant diffusion flux** at a timescale given by the latter; we denote $N_*(r)$ the number of stars within each radius r
→ # of stars evacuated from a given r with the above rate (per unit time) is constant (wrong argument):

$$N_*^2(r) \left(\frac{M_*}{M_{\text{BH}}} \right)^2 \frac{1}{P(r)} = \text{const.} \Rightarrow \rho_*(r) \propto r^{-9/4}$$

- **Bahcall & Wolf**: **Energy** $\sim M_*/r$ evacuated from a given r with the above rate (per unit time) **is constant** (correct argument):

$$\frac{N_*^2(r)}{r} \left(\frac{M_*}{M_{\text{BH}}} \right)^2 \frac{1}{P(r)} = \text{const.} \Rightarrow \rho_*(r) \propto r^{-7/4}$$

TDEs/EMRIs



- Initial position of a \star within the gray strip near the diag line (circular orbits $\rightarrow 45^\circ$ if properly scaled); they may **accidentally diffuse** around their j equilibria within the strip; some **refill the loss cone** \rightarrow diffuse “vertically” down to R_t
- \star with $r_{init} < R_{gw}$ diffuse “vertically” down **only to the red line** ($\sim r^{-1/2}$ dividing the “2-body region” from the “GW region”), then move “horizontally” to the **diagonal line**, and then **along it to R_t** : the ratio $\frac{\mathcal{R}_{TDE}}{\mathcal{R}_{EMRI}} = \frac{R_{inf}}{R_{gw}} = \left(\frac{R_t}{R_{schw}}\right)^2$

TDEs/EMRIs

- **Breakup of binaries** may alter this “picture” → “story” for a separate lecture
- For any **significantly eccentric orbit**, we do not need to change it much because the **pericenter is already close to BH**: the time-rate to change the pericenter is smaller by j^2 (where $j \sim \sqrt{r_p}$) → $\frac{1}{t} \sim N_*(r) \left(\frac{M_*}{M_{\text{BH}}} \right)^2 \frac{1}{P(r)} \frac{r_p}{r}$ → with $N_*(r)$ given from the BW cusp
- The timescale at which **GWS** can bring any orbit r to a smaller r_p :

$$T_{\text{gw}} = \frac{R_{\text{schw}}}{c} \frac{M_{\text{BH}}}{M_*} \left(\frac{r_p}{R_{\text{schw}}} \right)^4 \frac{r_p}{r} \left(\frac{r}{r_p} \right)^{3/2},$$

where the **1st fraction** is the timescale of **two equal bodies** at R_{schw} , the **2nd fraction** → **longer time** due to the **mass ratio**, the **3rd term** → the GR gives $t \sim r_p^4$ to shrink the orbit from r_p to R_{schw} , the **4th term** → the **lowering of emitted E** for much wider orbit $r > r_p$, and the **last one** → the longer time due to **lower velocity** at r

TDE energy budget

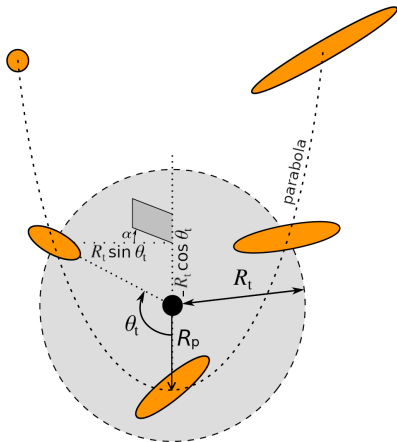
- Let's describe the basic dynamics associated with the **passage of the star at pericenter**:
- The star starts to be deformed at a few R_t from the BH \rightarrow stretched along an almost radial direction \rightarrow stellar debris **spread** with (specific) $E_{\text{orb}} = E = -GM_{\text{BH}}/(R_t + R_*)$:

$$E = E(R_t) + \Delta E \approx -\frac{GM_{\text{BH}}}{R_t} + \frac{GM_{\text{BH}}}{R_t^2} R_*$$

- At R_t , \star is **confined between two orbital planes** that intersect near pericentre, and \star is progressively **compressed into a "pancake"**

- We introduce β as R_t/R_p , in the limit $\beta \gg 1$, the **inclination angle between these two planes** is $\alpha \approx R_*/(R_t \sin \theta_t) \Rightarrow$ (parabola)

$$\cos \theta_t = \frac{2R_p}{R_t} - 1 \Rightarrow \sin \theta_t \approx \theta_t = \sqrt{\frac{R_p}{R_t}} \Rightarrow \alpha \approx \frac{R_*}{\sqrt{R_p R_t}}$$



Credit: Bonnerot+ 2017

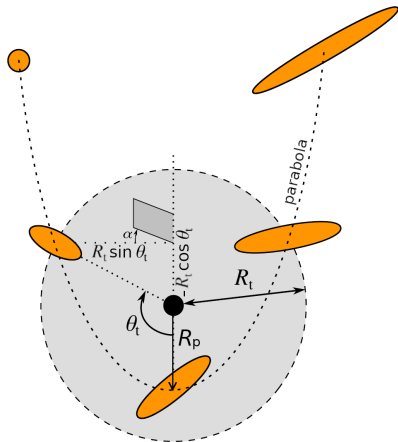
TDE energy budget

- At pericentre, in addition to shearing along the orbital plane, the star undergoes a **vertical compression** for a large β :

- The specific **vertical “compression” energy** $\Delta E_c = \frac{1}{2} \langle v_c^2 \rangle \Rightarrow$ the **pericenter “compression” velocity** $\sqrt{\langle v_c^2 \rangle} \approx \alpha v_p$ where $v_p = \sqrt{\frac{GM_{\text{BH}}}{R_p}}$ is the orbital velocity at R_p

- Substituting R_t and β , the latter gives

$$\langle v_c^2 \rangle \sim \Delta E_c = \beta^2 \frac{GM_\star}{R_\star}$$



Credit: Bonnerot+ 2017

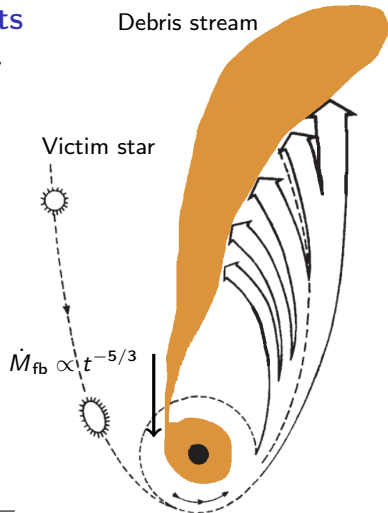
Fallback of matter on ballistic orbits

- About **half of the debris stream** produced by the disruption is bound to the BH \rightarrow it **comes back to the disruption site**
- The stream **moves ballistically** around BH \rightarrow the **most tightly bound debris** has an orbital energy $-\Delta E \approx -GM_{\text{BH}}/2a$ (Virial $\rightarrow E_k$ cancels half of E_p), which corresponds to a **semi-major axis** a_{min} and **orbital period** t_{min} (using the **Kepler law** $t^2 \propto a^3$ and assuming $dM/dE = \text{const.}$):

$$\frac{GM_{\text{BH}}}{R_{\star} \left(\frac{M_{\text{BH}}}{M_{\star}} \right)^{2/3}} = \frac{GM_{\text{BH}}}{2a} \Rightarrow$$

$$a_{\text{min}} = \frac{R_{\star}}{2} \left(\frac{M_{\text{BH}}}{M_{\star}} \right)^{2/3} \Rightarrow t_{\text{min}} \sim R_{\star}^{3/2} \frac{\sqrt{M_{\text{BH}}}}{M_{\star}} \Rightarrow$$

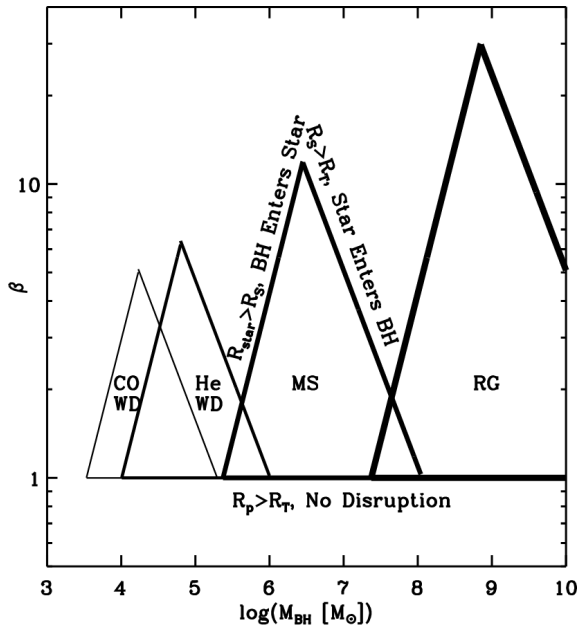
$$\Delta E \sim M_{\text{BH}}^{2/3} t^{-2/3} \rightarrow \dot{M} = \frac{dM}{dE} \frac{dE}{dt} \sim t^{-5/3}$$



- But: Stellar structure can modify dM/dE (Lodato+ 2009; Guillochon & Ramirez-Ruiz 2013)

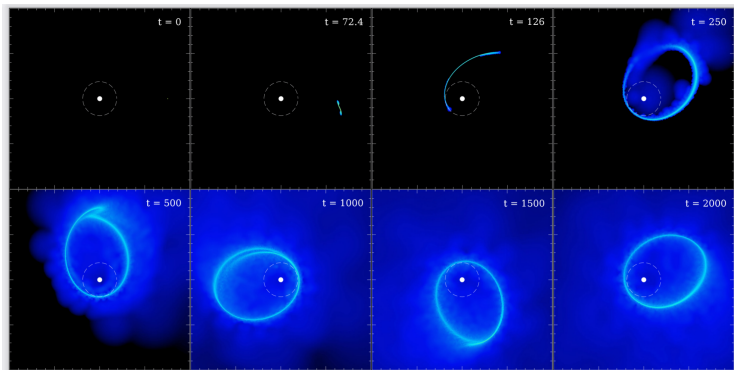
TDE models

- Allowable region for the TDEs of stars of different evolutionary states: a $0.6 M_{\odot}$ CO WD, a $0.17 M_{\odot}$ He WD, a $1 M_{\odot}$ MS star, and a $1.4 M_{\odot}$ RG, bounded by the conditions that $R_p < R_t$, $R_* < R_{\text{schw}}$, and $R_t > R_{\text{schw}}$ for a TDE to be observable, as a function of M_{BH} , and $\beta \equiv R_t/R_p$



Credit: Rosswog, Ramirez-Ruiz & Hix 2009

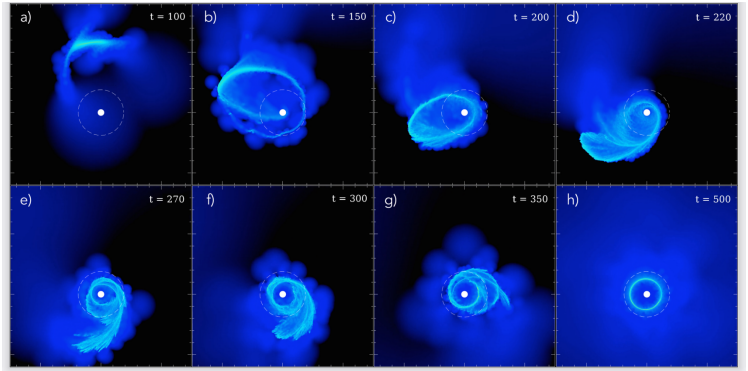
TDE models



Credit: Clerici & Gomboc 2020

- Two behaviours of the circularisation of the debris, depending on the initial orbital e and β : (a) shocks and higher precessing angles allow the debris to form a circular disc quickly and (b) the shocks are not impactful enough to allow fast circularisation \rightarrow the debris follows elliptical orbits \rightarrow Figure: the debris does not circularise efficiently and a disc is not formed (quickly)

TDE models



Credit: Clerici & Gomboc 2020

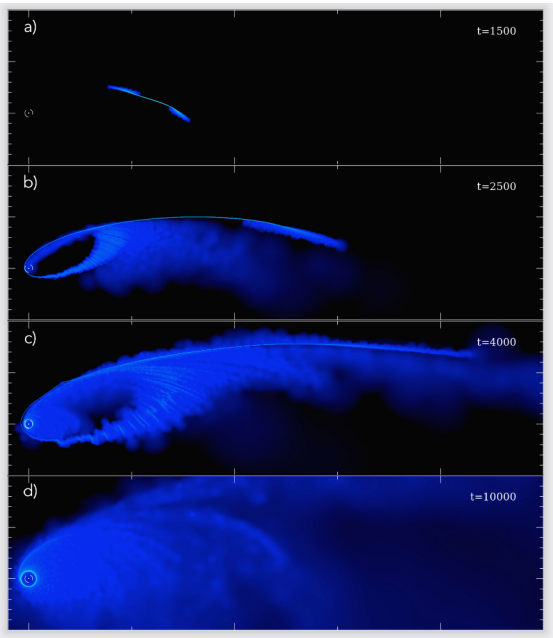
- **Circularisation of the stellar debris:** the debris quickly and efficiently circularises, mainly through self-crossings and shocks, and forms a disc with no debris falling back

TDE models

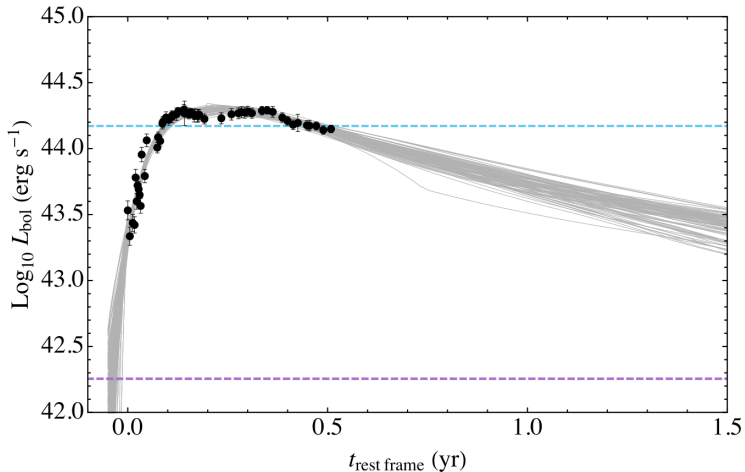
- **Circularisation of the stellar debris:** the debris circularises relatively quickly and forms a disc while there is still debris falling back

Credit:

Clerici & Gomboc 2020



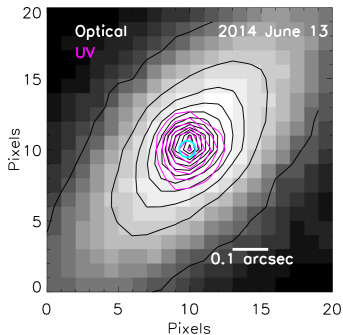
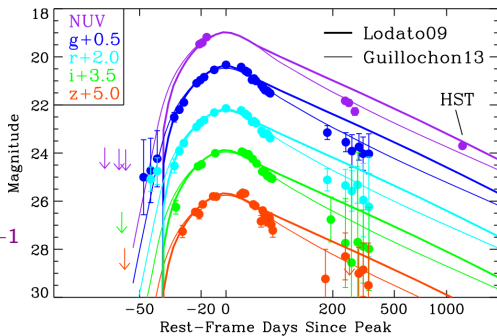
TDE models



- L_{bol} of TDE PS16dtm compared to an ensemble of models → dashed blue line shows L_{Edd} for fixed black hole mass of $10^6 M_{\odot}$, the dashed purple line shows the AGN luminosity prior to the flare → the projected behavior of the flare suggests that it will remain bright for years (Blanchard+ 2017)

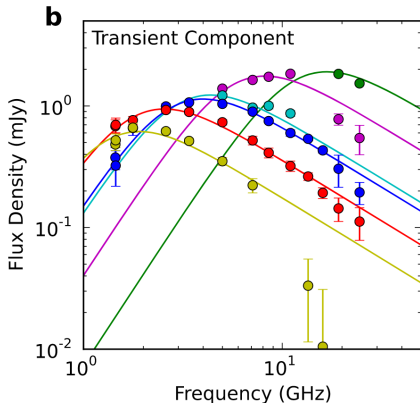
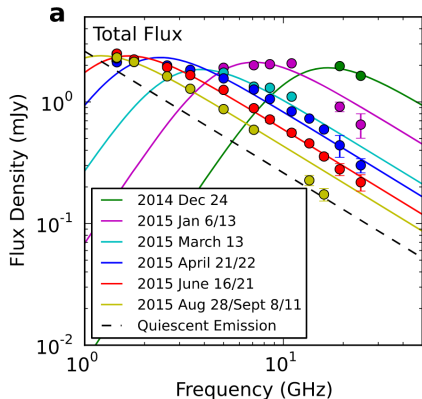
TDE observables

- Parameters of PS1-10jh:
- Peak luminosity: $\sim 10^{44} \text{ erg s}^{-1}$
- Temperature: $\sim 50\,000 \text{ K}$
- Line width: $\sim 5000 \text{ km s}^{-1}$
- Total energy: $\sim 10^{51} \text{ erg}$
- R_{em} : $\sim 10^{15} \text{ cm} = \sim 1000 R_{\text{Schw}}$



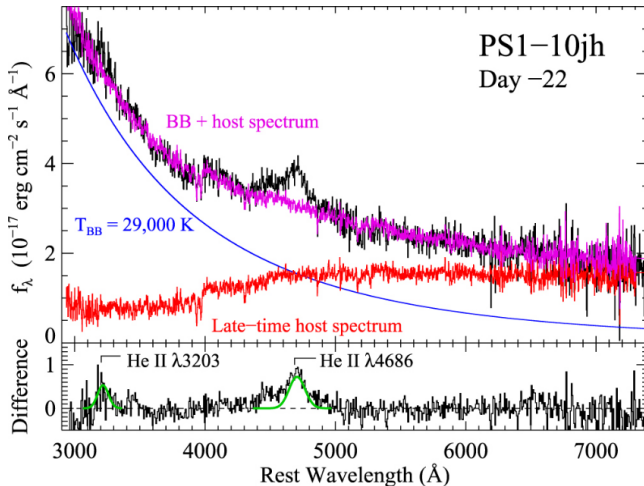
TDE observables

• Radio TDEs



Credit: Alexander et al. 2017

TDE observables



Credit: Gezari+ 2021

Detection of transient, broad He II line emission on top of the hot, thermal continuum 22 days before the peak of the flare in TDE PS1-10jh