Gravitational waves (GWs) - sources, binary mergers, common envelopes (CEs)

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Astronomical transients Selected chapters from astrophysics, fall semester, 2022

- Key work of Albert Einstein after the "Annus Mirabilis" 1905:
- Basics of the General Theory of Relativity (GTR) 1916
- Approximate integration of equations of the gravitational field (prediction of the gravitational waves) - 1916
- Cosmological considerations to the GTR 1917
- Previous important ideas and formulations:
- Principle of equivalence: acting of a (homogeneous) gravitational field and an accelerating frame are identical 1907
- Light bending and frequency shift in a gravitational field (determining the bending of a light beam by the gravitational field of the Sun) 1912
- Description of the relativistic theory of gravity based on the formalism of differential geometry (Carl Friedrich Gauss, Bernhard Riemann, Gregorio Ricci, Tullio Levi-Civita, Marcel Grossmann) 1913

- 1915: Einstein's equations of a gravitational field:
- Gesetz des Gravitationsfeldes Analogon der Poisson-Gleichung $\Delta\phi=4\pi G\rho$
- Im Materiefreien Fall: $R_{\mu\nu} = 0$
- Tensorgleichung statt skalarer, Tensordichte der Energie $T_{\mu\nu}$ statt Skalardichte ρ

•
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^2}T_{\mu\nu}$$

- matter distribution determines curvature of the spacetime
- curvature of spacetime drives the motion of matter

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•
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^2}T_{\mu\nu}$$

- Λ constant ightarrow stationary universe
- now we connect it with dark energy

•
$$\bar{h}_{ij}(\mathbf{r},t) \cong - \left. \frac{2G}{r} \frac{d^2 Q_{ij}}{dt^2} \right|_{t-|\mathbf{r}/c|}$$
 gravitational quadrupole perturbation

Über Gravitationswellen.

Von A. EINSTEIN.

(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

Die wichtige Frage, wie die Ausbreitung der Gravitationsfelder erfolgt, ist schon vor anderthalb Jahren in einer Akademiearbeit von mir behandelt worden¹. Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen.

Wie damals beschränke ich mich auch hier auf den Fall, daß das betrachtete zeiträumliche Kontinuum sich von einem «galileischen« nur sehr wenig unterscheidet. Um für alle Indizes

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \tag{1}$$

setzen zu können, wählen wir, wie es in der speziellen Relativitätstheorie üblich ist, die Zeitvariable x_i rein imaginär, indem wir

156 Gesamtsitzung vom 14. Februar 1918. - Mitteilung vom 31. Januar

so gewählt werden, daß die $g_{\mu\nu}$ des neuen Systems vier willkürlich vorgeschriebenen Beziehungen genügen. Diese denken wir so gewählt, daß sie im Falle der uns interessierenden Näherung in die Gleichungen (5) übergehen. Die letzteren Gleichungen bedeuten also eine von uns gewählte Vorschrift, nach welcher das Koordinatensystem zu wählen ist. Vermöge (5) erhält man an Stelle von (4) die einfachen Gleichungen

$$\sum_{\alpha} \frac{\partial^2 \gamma'_{\mu\nu}}{\partial x^2_{\alpha}} = 2 \times T_{\mu\nu}.$$
 (6)

Aus (6) erkennt man, daß sich die Gravitationsfelder mit Lichtgeschwindigkeit ausbreiten. Die $\gamma_{\mu\nu}$ lassen sich bei gegebenen $T_{\mu\nu}$ aus letzteren nach Art der retardierten Potentiale berechnen. Sind x, y, z, t die reellen Koordinaten $x_i, x_2, x_3, \frac{x_4}{i}$ des Aufpunktes, für welchen die $\gamma'_{\mu\nu}$ berechnet werden sollen, x_o, y_o, z_o die räumlichen Koordinaten eines Raumelementes dV_o, r der räumliche Abstand zwischen letzterem und dem Aufpunkt, so hat man

$$\gamma'_{uv} = -\frac{\kappa}{2\pi} \int \frac{T_{uv}(x_o, y_o, z_o, t-r)}{r} \, d\, V_o \,. \tag{7}$$

But his later work showed much confusion!

Einstein & Rosen 1936, Physical Review (submitted): Claimed gravitational waves do not exist! (paper rejected by

H. P. Robertson from Caltech)

ON GRAVITATIONAL WAVES.

BY

A. EINSTEIN and N. ROSEN.

ABSTRACT.

The rigorous solution for cylindrical gravitational waves is given. For the convenience of the reader the theory of gravitational waves and their production, already known in principle, is given in the first part of this paper. After encountering relationships which cast doubt on the existence of rigorous solutions for undulatory gravitational fields, we investigate figurously the case of cylindrical gravitational waves. It turns out that rigorous solutions exist and that the problem reduces to the usual cylindrical waves in euclidean space.

I. APPROXIMATE SOLUTION OF THE PROBLEM OF PLANE WAVES AND THE PRODUCTION OF GRAVITATIONAL WAVES.

It is well known that the approximate method of integration of the gravitational equations of the general relativity theory leads to the existence of gravitational waves. The method used is as follows: We start with the equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -T_{\mu\nu}.$$
 (1)

We consider that the g_{μ} , are replaced by the expressions

$$g_{\mu\nu} = \delta_{\mu\nu} + \gamma_{\mu\nu}, \qquad (2)$$

Einstein & Rosen 1937, Journal of Franklin Institute, 223, 43:
 Oops! Gravitational waves actually do exist! (after correction of a bad choice of coordinates from the 1936 paper)

• Einstein 1939, Annals of Mathemathics 40, 922:

"Proof" that black holes cannot exist in nature

Have an environment with each of the environment o

vanishes for $r = \mu/2$. This means that a clock kept at this place would go at the rate zero. Further it is easy to show that both light rays and material particles take an infinitely long time (measured in "coordinate time") in order to reach the point $r = \mu/2$ when originating from a point $r > \mu/2$. In this sense the spherer $r = \mu/2$ constitutes a place where the field is singular. (μ represents the gravitating mass.)

There arises the question whether it is possible to build up a field containing such singularities with the help of actual gravitating masses, or whether such regions with vanishing $g_{\rm st}$ do not exist in cases which have physical reality. Schwarzschild himself investigated the gravitational field which is produced by an incompressible liquid. He found that in this case, too, there appears a region with vanishing $g_{\rm st}$ f only, with given density of the liquid, the radius of the field conduction moders in dimensioned here second.

= "proof" that even genius can be wrong ...

Proof of the gravitational waves existence

Hulse-Taylor pulsar (PSR B1913+16) observed since 1974 \rightarrow gravitational waves carry away energy and momentum. Nobel prize 1993. It is expected that in \sim 300 Myrs both components merge.



Gravity waves detector: Advanced LIGO

LIGO (Laser Interferometer Gravitational-Wave Observatory) consists of two identical distant interferometers with length of the "arms" 4 km - 1st detection: GW150914

Advanced LIGO Fabry-Perot Michelson Interferometer Schematic



Detector of gravitational waves LIGO (\rightarrow advanced LIGO)

First detection of GW 150914 with LIGO



Future prospect: LIGO-India, Cosmic Explorer LISA,...

LISA (Laser Interferometer Space Antenna) observatory is the planned ESA project \rightarrow each of the three satellites orbit around the Sun and distant mutually of about $10^6\,km$ \rightarrow planned launch in 2034



Masses in the Stellar Graveyard ? ? 0 0 0 0 0 EM Neutron Stars IGO-Virgo Neutron Stars

LIGO/VIRGO gravitational waves detections

• Why do we need three (or more) detectors:



LIGO/VIRGO detection of GW 170817 - first EM counterpart





Merger rates from LIGO/VIRGO events and their total estimations

- LIGO/VIRGO BH-BH merger rate (17 confirmed events; still depends on ill-determined mass distribution; BH remnants)
 12 - 200 Gpc⁻³ yr⁻¹
- LIGO/VIRGO NS-NS merger rate (2 confirmed events; very important due to connection to EM counterparts; NS remnant in one case / ? in the other one)
 300 4000 Gpc⁻³ yr⁻¹
- LIGO/VIRGO BH-NS or BH-? merger rate (3 confirmed events (BH remnants)
- compare CCSN rate (at least for the low z universe)
 0 10⁵ Gpc⁻³ yr⁻¹

Stellar initial mass function (IMF) $dN_{\star,init}/dM$

- old Salpeter IMF: most M (weakly) in BDs and RDs (now known incorrect) $\frac{dN_{\star}}{dM} \propto M^{-2.35} \quad 0.08 < M < 100 M_{\odot}$
- Chabrier IMF (similar Kroupa or Parravano): most M in 0.5 1 M_{\odot} stars $\frac{dN_{\star}}{dM} \propto M^{-1} \exp\left[-\frac{1}{2}\left(\frac{\log M - \log 0.079}{0.69}\right)^{2}\right] \qquad M < M_{\odot}$ $\frac{dN_{\star}}{dM} \propto M^{-2.3} \qquad 1 < M < 100 M_{\odot} \qquad 10$
- Density fluctuations and high Mach number turbulence in molecular clouds produce the universal IMF spectrum (see also Guszejnov, Krumholz, & Hopkins 2016)



Credit: Krumholz & Federrath 2019

Chabrier IMF

- For every 1 M_☉ mass of stars formed according to Chabrier IMF:
 - 0.46 M_{\odot} of 0.01 1 M_{\odot} stars form
 - 0.44 M_{\odot} of 1-50 M_{\odot} stars form
 - Most by # are BDs/RDs (<0.3 M_☉)

- For every 1 M_☉ mass of gas converted to stars according to Chabrier IMF:
 - $\bullet~2.99$ stars 0.01 50 M_{\odot}
 - 0.48 stars 0.3 1 M_{\odot}
 - 0.14 stars 1-3 M_{\odot}
 - 0.033 stars 3 10 M_{\odot}
 - 0.0053 stars 10 20 M_{\odot}
 - 0.0025 stars 20-50 M_{\odot}

• MW today:

- stars: $5.9 \times 10^{10} M_{\odot}$ (mostly old <1 M_{\odot}), gas disk: $5 \times 10^9 M_{\odot}$ (exhausted in 3 Gyr by SFR = 1.6 M_{\odot} yr⁻¹: infall/fountain!)
- CC SNe from 10-20 M_{\odot} , rate = 1.6 M_{\odot} yr⁻¹*0.0053/ M_{\odot} : # = 1/120 yr
 - youngest known CC SNR: G1.9+0.3 with age 110 yr
 - second youngest known: Cas A with age ${\sim}300\,\text{yr}$
- # of NS (if from 10-20 M_{\odot}): 5.9×10¹⁰ $M_{\odot}/0.46 \times 0.0053 = 7 \times 10^8$!

Extrapolation of the MW rates to the local universe

• MW:

- SFR = 1.6(2) M_{\odot} /yr (Kroupa IMF)
- $M_{\rm bulge} = 0.9(1) \times 10^{10} M_{\odot}$
- $M_{\rm disk} = 5(1) \times 10^{10} \ M_{\odot}$

(Licquia & Newman 2015)

- young stars scaled by current
 SFR: = MW rate * 0.016/Mpc³
- rate for middle aged stars scaled by blue light: = MW rate * 0.01/Mpc³ (Phinney 1991)

z = 0 universe:

- SFR density = 0.025(2) $M_{\odot}/\text{Mpc}^3/\text{yr}$
- of which 20% in starbursts

(Bothwell+ 2011 using IR+UV; Kennicutt+ 2021)

• stellar mass density = $3.2 \times 10^8 M_{\odot} \text{ Mpc}^{-3}$

(Cole+ 2001 using J, K, IR LF; Karachentsev & Telikova 2018)

scaled by stellar mass rate (including elliptical galaxies, etc.): = MW rate * 0.005/Mpc³

SFR scaling with redshift z

- SFR(z) $\equiv \psi(z) = 0.013 \frac{(1+z)^{2.7}}{1 + [(1+z)/2.9]^{5.6}} M_{\odot} \text{ Mpc}^{-3} \text{ yr}^{-1} \Rightarrow \text{ peaks at } \sim 9$ at $z \sim 2$ (Madau & Dickinson 2014)
- Rate of CCNS = 0.01 $M_{\odot}^{-1}\psi(z)$
- Formation scenarios:
 - $t_{\rm mrg} (1.4 + 1.4 \, M_{\odot}) = 10^{10} \, {\rm yr}$ for $a = 4 \, R_{\odot}$, $P_{\rm orb} = 0.6 \, {\rm d}$
 - $t_{\rm mrg} (30 + 30 M_{\odot}) = 10^{10} \, {\rm yr}$ for $a = 43 R_{\odot}$, $P_{\rm orb} = 4 \, {\rm d}$
 - Evolution must end up with close binary BH-BH, BH-NS, or NS-NS so as to merge through gravitational radiation in 10¹⁰ yr
 - In traditional (fusion in core which does not mix with envelope) stellar evolution, massive stars that produce NS, BH, swell to very large radii ${\sim}1000\,R_{\odot}$
 - Thus *in the field*, merging compact object binaries require CE evolution

Crucial role of CEs

• Formation of compact binary with BH or NS components

 The post-CE evolution shows two different possible channels leading to completely different outcomes

Credit: Postnov & Yungelson 2016



Crucial role of CEs

- CE scenario leading to a BH-BH merger similar to GW150914
- Z=0.0006 (1/30 Z_☉)
- Start at $z \sim 0.32$ (2 Gyr after BB, end at z = 0.09 (distance ~ 0.45 Gpc)
- The separation shrinks at a 100 during the CE phase!
- HG: Hertzsprung-gap star;
 CHeB: core-He-burning star

Credit: Belczynski+ 2016



Homogeneous chemical evolution

- Scenario for GW 150914
- Z=0.0004 (1/50 Z_☉)
- Note very little change in orbital separation during evolution!

Credit: Marchant+ 2016, see also de Mink & Mandel 2016



Final configurations of massive binaries

- Total masses and orbital periods at core He depletion for systems with $M_1/M_2 = 1$ at four different metallicities
- Dashed lines show constant merger times assuming direct collapse into a BH
- The shaded region indicates the mass range at which **PISNe would occur**
- The coloured bands represent the relative number of objects formed for each Z



- Kozai Lidov cycles in field triples:
- Close to $a_2/a_1 \sim 10$, near equal mass triples can via Kozai cycles push inner binary to very high eccentricity and rapid merger
- Field rate (if no natal kicks) ~ 6 Gpc⁻³ yr⁻¹ (< low end of current LIGO estimate)
- Much lower with natal kicks of even 40 km s⁻¹ :median of Galactic black hole binaries (Silsbee & Tremaine 2017)



Credit: DPA UCLA

- Prospects for observing the formation scenarios other galaxies:
- BHs: 1/yr at \sim 150 Mpc \rightarrow all sky
- NSs: 1/yr at \sim 70 Mpc \rightarrow all sky
- CEs: BH or NS: very long, slow (yrs & decades) IR CEs; possibly with high Γ jets with accretion to distinguish from more common MS stars inspiralling
- X-ray/UV TDEs from WDs disrupted by BH in GCs
- Extragalactic SS433s?

- Prospects for testing the formation scenarios MW + friends:
- GCs: search with PFs (or similar equipments) for other numerous hard $MS + 30 M_{\odot}$ BH binaries in every cluster predicted by dynamical formation models
- Continue hunt for failed CEs = Thorne Żytkow objects (p-process \rightarrow p-nuclei)
- Search for pre-merger Kozai BH-BH + MS (Silsbee & Tremaine) or pre-merger hierarchical BH-BH + MS (Wen & Phinney) systems, including among ULXBs
- Better evidence for or against rotational mixing in close (low Z?) binaries
- BH masses in BH transients: GAIA astrometric binaries, IR orbits for obscured low-kick quiescent X-ray transients(cf. Junker program)
- Use VLBI microlensing to measure mass function of galactic single BHs (cf. Karami+ 2016)

- Binary BHs in triples with accreting companions
- A stable hierarchical triple system of massive stars (commonly formed by ZAMS in Galactic disk; also dynamical exchange in GCs)
- Inner binary evolves the same way as lone binary progenitors for compact binary BHs
- Inner binary forms binary BHs the same way as lone binaries; if kicks are not too large and not huge mass loss, 3rd stars survives the process of forming BHs
- After forming binary BHs; 3rd companion starts to fill its Roche lobe, accreting onto binary BHs; looking like bright X-ray binary sources; accretion can help drive BBH to merge faster
- Formation of circumbinary disk, super-Eddington accretion at binary merger, shock-heated disk at merger
- Possible EM counterparts in X-ray, optical, and radio for BHB merger in LIGO band

Other proposals for getting EM from circum/intra binary disks around BH-BH

- Cold remnant circumbinary disks (from stellar mass transfer) reactivated by BH-BH merger recoil and mass loss (de Mink & King 2017 + the previous ideas for SMBH binaries, cf. Rossi+ 2010, Milisavljevic & Phinney 2005)
- Binary BHs formed/captured in the dense AGN disk hardening by 3-body and gas drag shrink the orbit to merge, accretion from surrounding dense AGN disk (Stone, Metzger & Haiman 2017: rate 0.1-3 Gpc⁻³ yr⁻¹; optimistically)
- "Frozen" (neutral, MRI off) reactivated by tidal torques as BH-BH approach merger (Perna+ 2016, Kimura+ 2016)
- In all cases probably just an ultra-luminous X-ray source

Evolution of binary stars orbit

• Let's now consider the evolution of a binary star on a circular orbit in the xy-plane. The stars have masses m_1 and m_2 and separation a: They orbit each other with an angular frequency ω and orbital energy E_{orb} :

$$\omega = \sqrt{\frac{G(m_1 + m_2)}{a^3}}, \qquad E_{\rm orb} = -\frac{Gm_1m_2}{2a}$$

• The quadrupole moment is $(2\cos^2\omega t - \sin 2\omega t - 0)$

$$Q = \frac{1}{2}\mu a^2 \begin{pmatrix} 2\cos\omega t & \sin 2\omega t & 0\\ \sin 2\omega t & 2\sin^2\omega t & 0\\ 0 & 0 & 0 \end{pmatrix}$$
$$\ddot{Q} = 4\mu a^2 \omega^3 \begin{pmatrix} \sin 2\omega t & -\cos 2\omega t & 0\\ -\cos 2\omega t & -\sin 2\omega t & 0\\ 0 & 0 & 0 \end{pmatrix}$$

• GW "luminosity": $L_{GW} = \frac{G}{5c^5} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} - \frac{1}{3} \ddot{Q}_{ii} \ddot{Q}_{jj} \rangle = \frac{32G\mu^2 a^4 \omega^6}{5c^5}$, which gives $\sim 10^{45} \text{ erg s}^{-1}$ for $1 M_{\odot}$ star orbiting around Galactic SMBH at R_{schw} , $\sim 10^{40} \text{ erg s}^{-1}$ for two $30 M_{\odot}$ BHs orbiting at a distance Earth - Moon, and $\sim 10^{57} \text{ erg s}^{-1}$ for two $30 M_{\odot}$ BHs orbiting at a distance of touching their R_{schw} 's

Evolution of binary stars orbit

• Setting
$$-\dot{E} = -Gm_1m_2\dot{a}/2a^2$$
 and evaluating \dot{a} , we get

$$\frac{1}{4}a^4 = \frac{1}{4}a_{init}^4 - \frac{64G^3\mu^2(m_1 + m_2)^3}{5m_1m_2c^5}t$$

• The stars must merge in a gravitational wave inspiral time:

$$t_{\rm GW} = \frac{5c^5(m_1 + m_2)^{1/3}}{256G^{5/3}m_1m_2\omega_{\rm init}^{8/3}},$$

which for two NSs, $m_1=m_2=1.4\,M_\odot$, leads to a merger in $t_{\rm GW}=10\,{\rm Gyr}$ if $P_{\rm init}=2\pi/\omega_{\rm init}=15\,{\rm hr}$

• For the mergers seen with LIGO we define the "chirp" frequency that increases right before the merger and get a constraint on the combination of the masses $(m_1 + m_2)^{1/3}/m_1m_2$ where we define the chirp mass

$$M_{
m chirp} \equiv \left[rac{m_1 m_2}{(m_1 + m_2)^{1/3}}
ight]^{3/5}$$

*M*_{chirp} is the best-constrained property of a LIGO event → measurement of the individual masses → higher-order relativistic corrections → not as well constrained

NS-NS ejecta power from accretion (sGRB)

- The rest energy of the Sun:
 - $M_{\odot}c^2 \approx 2 \times 10^{54} \, {\rm ergs}$



- Two closely orbiting NS with *not quite equal masses* + *being not too compact*; they'll tidally distort each other and the material on the far side of the tidal bulges can become unbound: **spilling then into tidal tails**
 - GR simulations indicate $\Delta M_{
 m tidal} pprox 10^{-2} \, M_{\odot}$
- As the stars touch each other → collisional shocks that spray out the hot material in or near the contact plane:
 - mass spreaded $\rightarrow \Delta M_{\rm ej} \approx 10^{-2} M_{\odot}$
- This stuff is ejected with high velocities; probably not fully escaping the whole system; the tidal streamers eventually intersect each other, etc., making a disk and accreting & falling back onto the BH
 - Assuming roughly the 10% efficiency, the accretion energy might be about $\Delta E_{accr} = \Delta (M_{ej} + M_{tidal})c^2 \times 0.1 \approx 10^{51} \text{ ergs} = \text{nice } \mathbf{GRB}$