Hledáme hranice Vesmíru

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15. dubna 2025, Brno, Univerzita třetího věku

844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

25. listopadu 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der »Materie« verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß $\sqrt{-g}$ zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren. Dabei mußte aber, wie erwähnt, die Hypothese eingeführt werden, daß der Skalar des Energietensors der Materie verschwinde.

Neuerdings finde ich nun, daß man ohne Hypothese über den Energietensor der Materie auskommen kann, wenn man den Energietensor der Materie in etwas anderer Weise in die Feldgleichungen einsetzt, als dies in meinen beiden früheren Mitteilungen geschehen ist. Die Feldgleichungen für das Vakuum, auf welche ich die Erklärung der Perihelbewegung des Merkur gegründet habe, bleiben von [2]
dieser Modifikation unberührt. Ich gebe hier nochmals die ganze Betrachtung, damit der Leser nicht genötigt ist, die früheren Mitteilungen unausgesetzt heranzuziehen.

Aus der bekannten Riemannschen Kovariante vierten Ranges leitet man folgende Kovariante zweiten Ranges ab:

$$G_{im} = R_{im} + S_{im} \tag{1}$$

$$R_{im} = -\sum_l \frac{\partial \begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l} + \sum_{lt} \begin{Bmatrix} il \\ \rho \end{Bmatrix} \begin{Bmatrix} m\rho \\ l \end{Bmatrix} \tag{1a}$$

$$S_{im} = \sum_l \frac{\partial \begin{Bmatrix} il \\ l \end{Bmatrix}}{\partial x_m} - \sum_{lt} \begin{Bmatrix} im \\ \rho \end{Bmatrix} \begin{Bmatrix} \rho l \\ l \end{Bmatrix} \tag{1b}$$

[1] Sitzungsber. XLIV, S. 778 und XLVI, S. 799, 1915. [1]

Die allgemein kovarianten zehn Gleichungen des Gravitationsfeldes in Räumen, in denen »Materie« fehlt, erhalten wir, indem wir ansetzen

$$G_{im} = 0. \tag{2}$$

Diese Gleichungen lassen sich einfacher gestalten, wenn man das Bezugssystem so wählt, daß $\sqrt{-g} = 1$ ist. Dann verschwindet S_{im} wegen (1b), so daß man statt (2) erhält

$$R_{im} = \sum_l \frac{\partial \Gamma^l_{im}}{\partial x_l} + \sum_{lt} \Gamma^l_{il} \Gamma^t_{ml} = 0 \tag{3}$$

$$\sqrt{-g} = 1. \tag{3a}$$

Dabei ist

$$\Gamma^l_{im} = -\begin{Bmatrix} im \\ l \end{Bmatrix} \tag{4}$$

gesetzt, welche Größen wir als die »Komponenten« des Gravitationsfeldes bezeichnen.

Ist in dem betrachteten Raume »Materie« vorhanden, so tritt deren Energietensor auf der rechten Seite von (2) bzw. (3) auf. Wir setzen

$$\boxed{G_{im} = -\varkappa \left(T_{im} - \frac{1}{2} g_{im} T \right),} \tag{2a}$$

wobei

$$\sum_{tr} g^{tr} T_{tr} = \sum_r T^r_r = T \tag{5}$$

gesetzt ist; T ist der Skalar des Energietensors der »Materie«, die rechte Seite von (2a) ein Tensor. Spezialisieren wir wieder das Koordinatensystem in der gewohnten Weise, so erhalten wir an Stelle von (2a) die äquivalenten Gleichungen

$$R_{im} = \sum_l \frac{\partial \Gamma^l_{im}}{\partial x_l} + \sum_{lt} \Gamma^l_{il} \Gamma^t_{ml} = -\varkappa \left(T_{im} - \frac{1}{2} g_{im} T \right) \tag{6}$$

$$\sqrt{-g} = 1. \tag{3a}$$

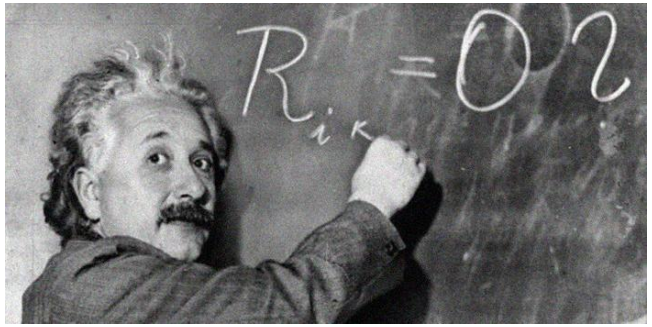
Wie stets nehmen wir an, daß die Divergenz des Energietensors der Materie im Sinne des allgemeinen Differentialkalkuls verschwinde (Impulsenergiesatz). Bei der Spezialisierung der Koordinatenwahl gemäß (3a) kommt dies darauf hinaus, daß die T_{im} die Bedingungen

$$\sum_\lambda \frac{\partial T^\lambda_r}{\partial x_\lambda} = -\frac{1}{2} \sum_{\mu\nu} \frac{\partial g^{\mu\nu}}{\partial x_r} T_{\mu\nu} \tag{7}$$

oder

$$\sum_\lambda \frac{\partial T^\lambda_r}{\partial x_\lambda} = -\sum_{\mu\nu} \Gamma^\mu_{r\nu} T^\nu_\mu \tag{7a}$$

erfüllen sollen.

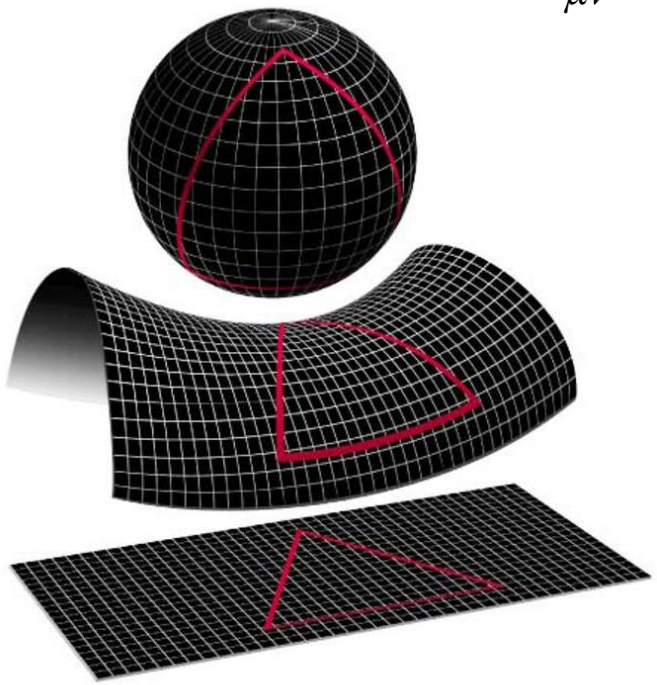


A. Einstein (1879 – 1955)

Geometrie časoprostoru = zdroje deformace

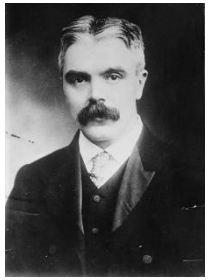
$$\boxed{R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}}$$

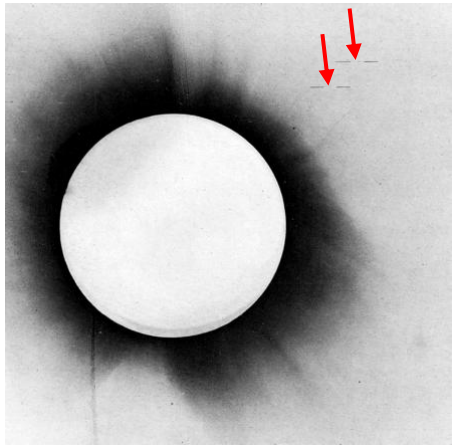
$$\approx 10^{-41} T_{\mu\nu}$$



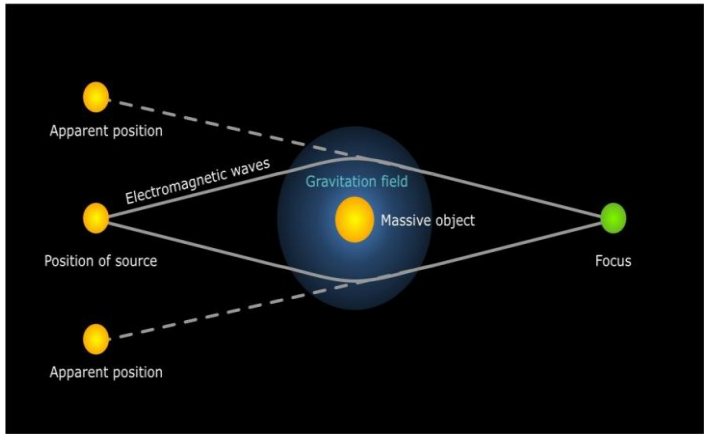
Obecná teorie relativity

Úplné zatmění Slunce (1919)



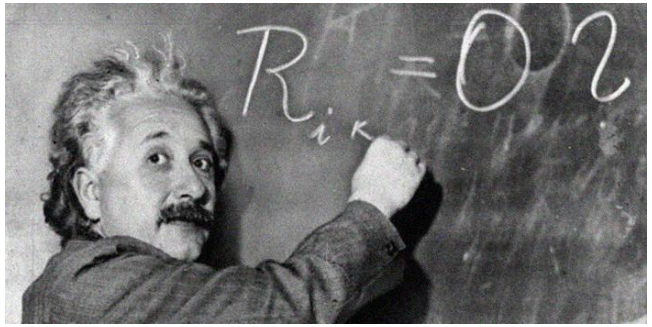


A. Eddington (1882 – 1944)
F. W. Dyson (1868 – 1939)



Gravitační čočka

Einsteinův kříž



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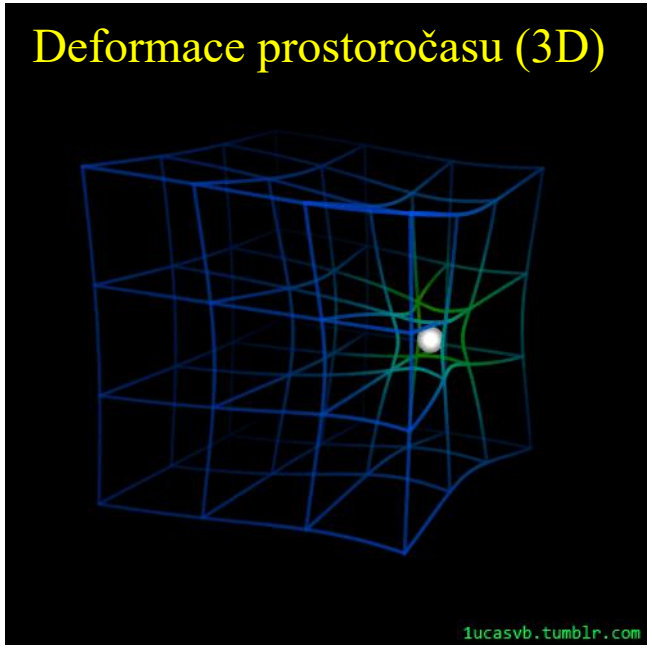

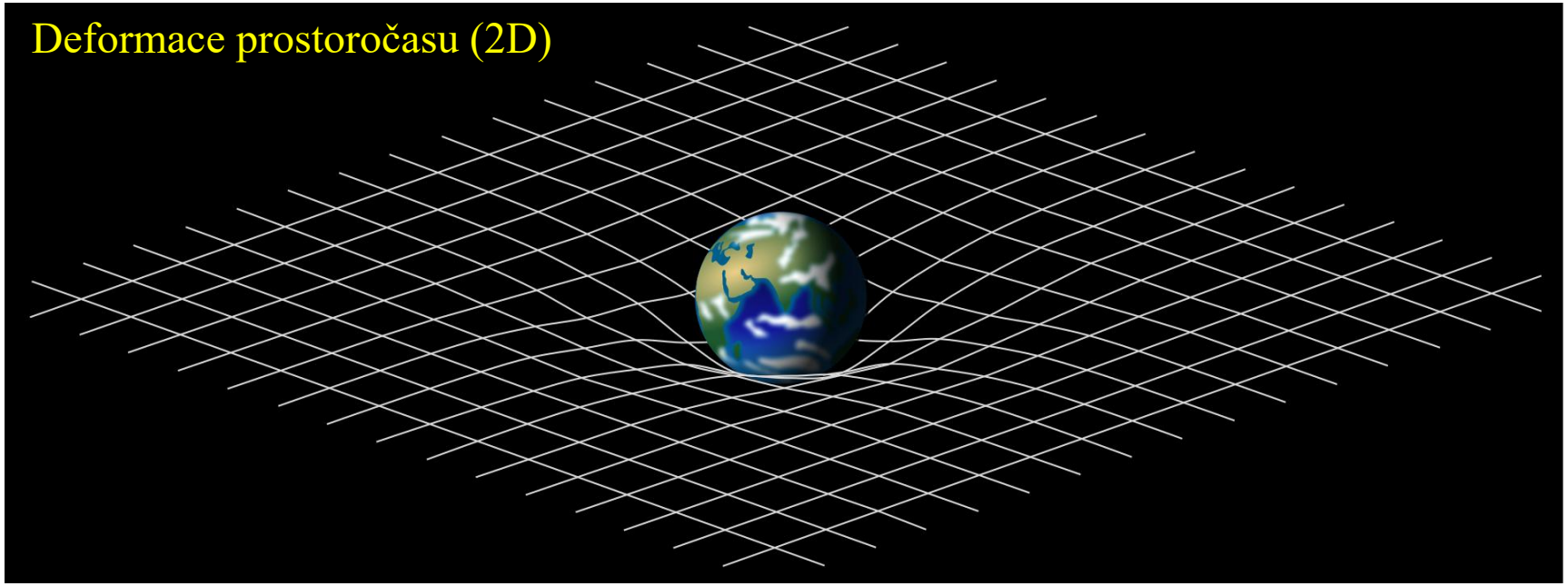

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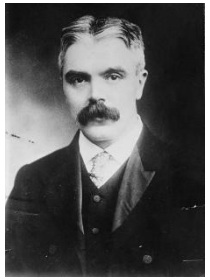

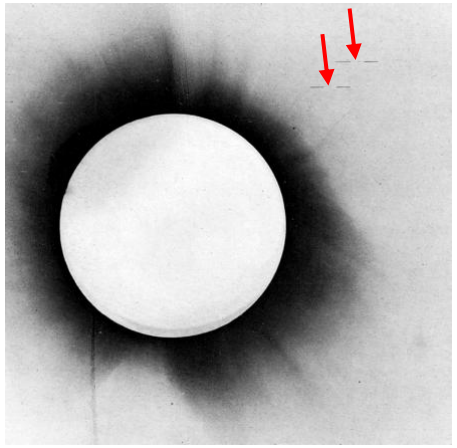

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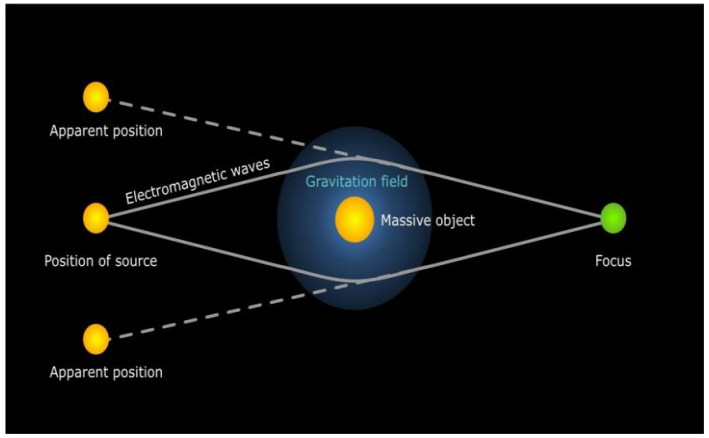

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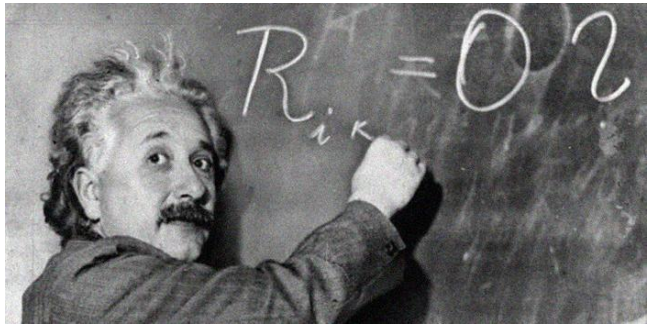

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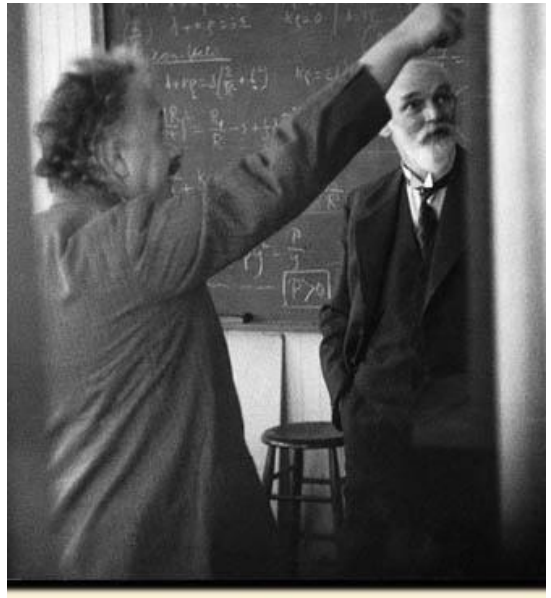

- 1917 A. Einstein – matematický model vesmíru
- 1917 W. de Sitter – rozpínající se Vesmír bez hmoty
- 1922 A. A. Friedmann – Vesmír s hmotou
- a další…

A. A. Friedmann
(1888 – 1925)
Алекса́ндр Алекса́ндрович Фри́дман

Obecná teorie relativity

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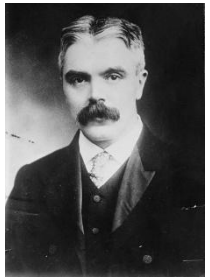

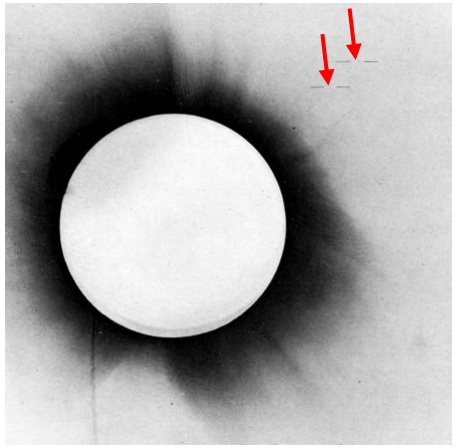

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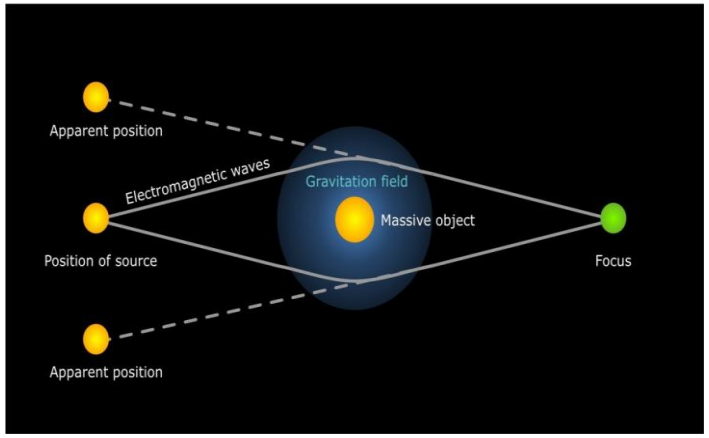

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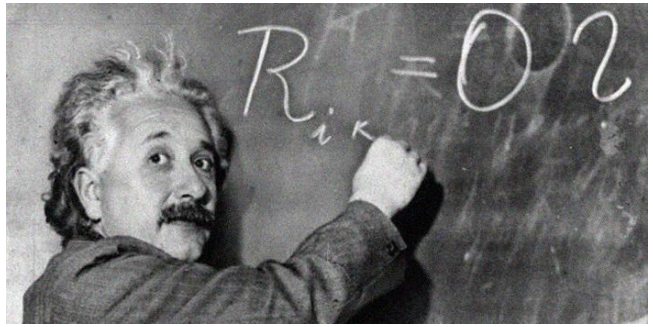

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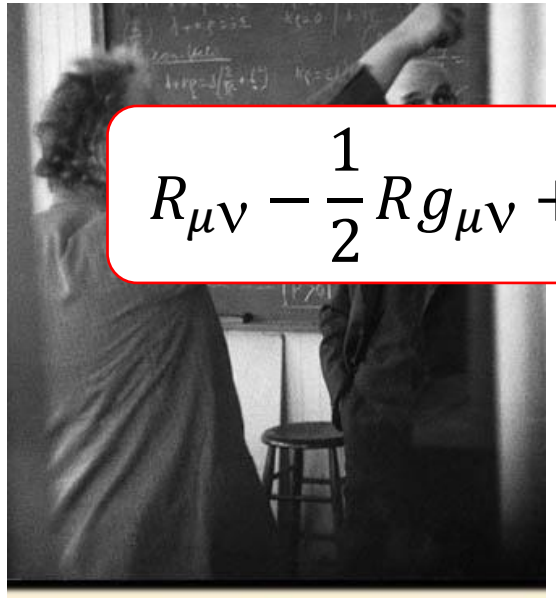


$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

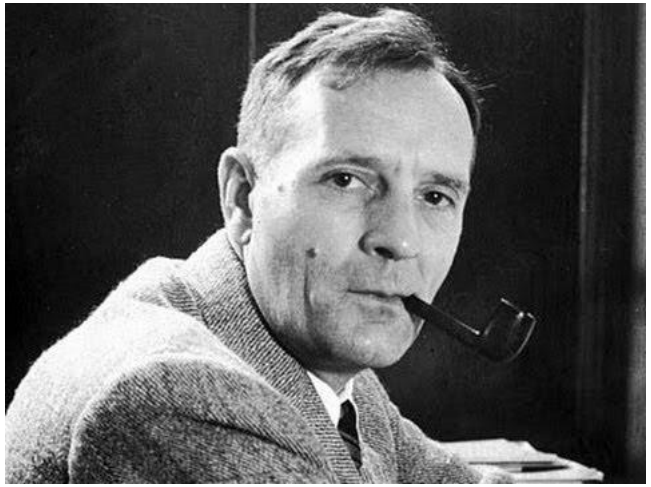
Kosmologická konstanta

Nestabilní Vesmír

A. A. Fridman
(1888 – 1925)

Rozpínající Vesmír

1 Mpc = 3,26 Mly
1 ly = 9,46 × 10^{12} km



E. Hubble (1889 – 1953)

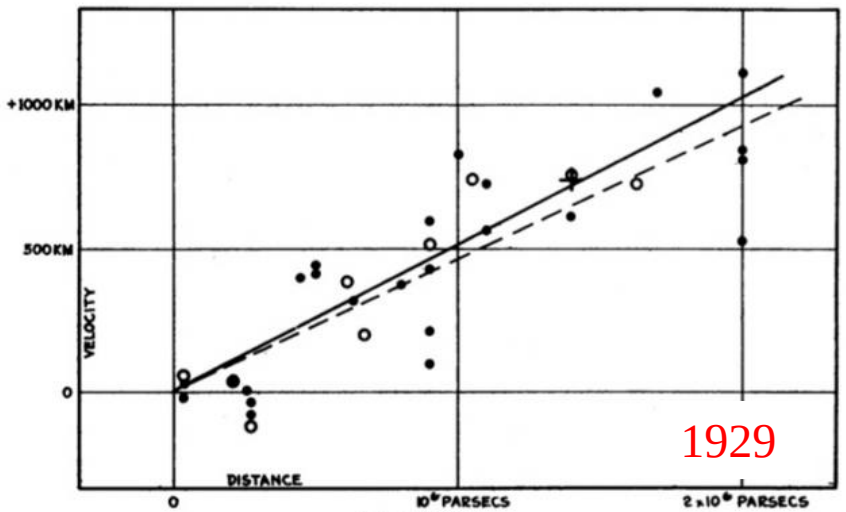
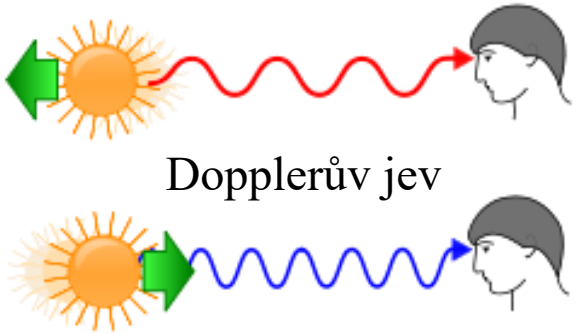


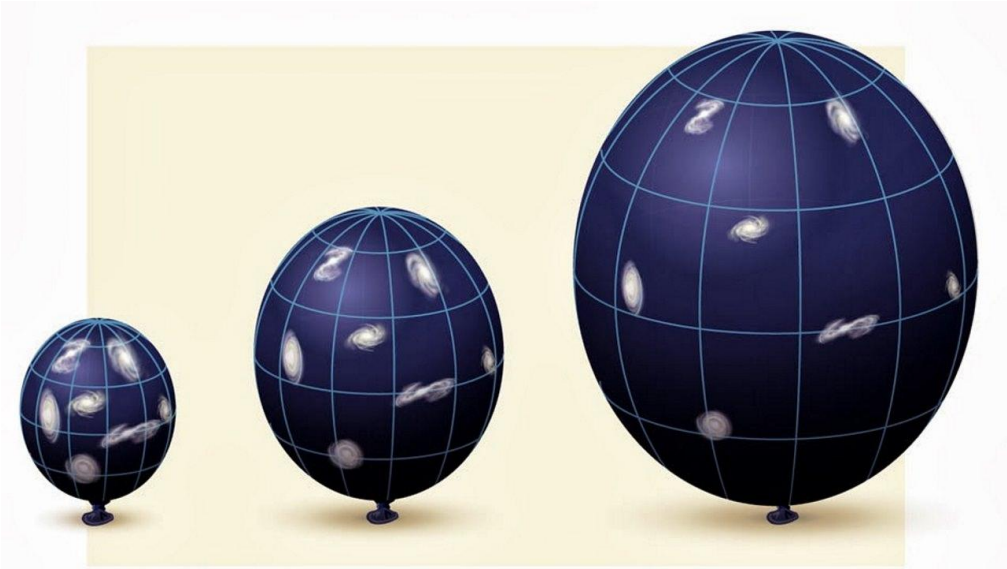
FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

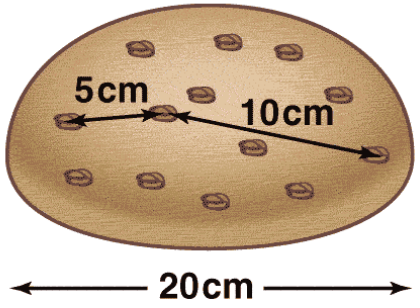


Hubbleova konstanta:

$$v = HR(t)$$

$$H = \frac{\dot{R}(t)}{R(t)}$$





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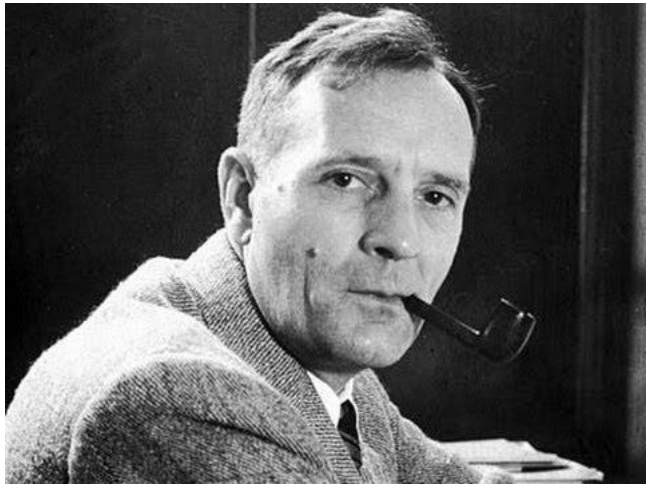

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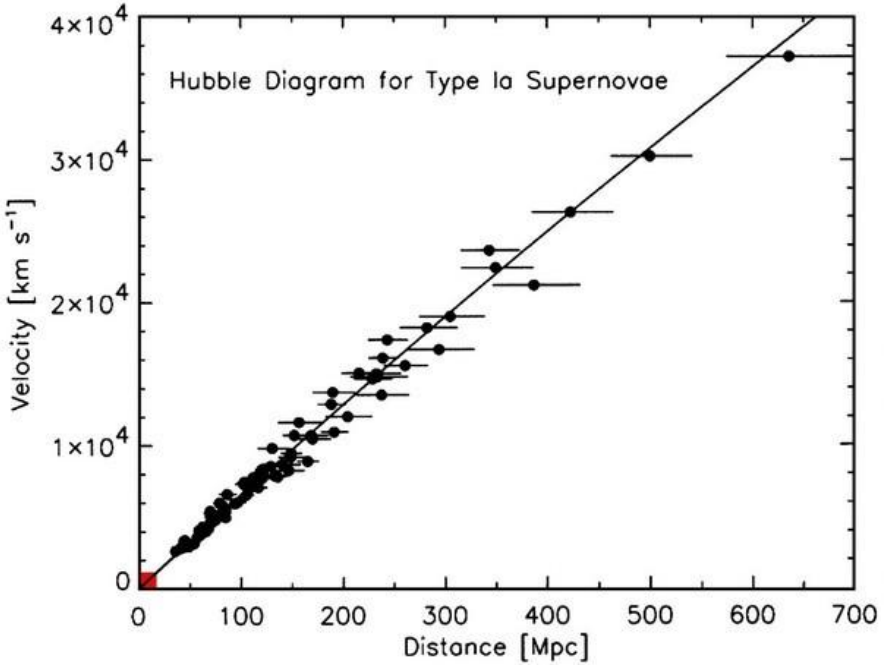

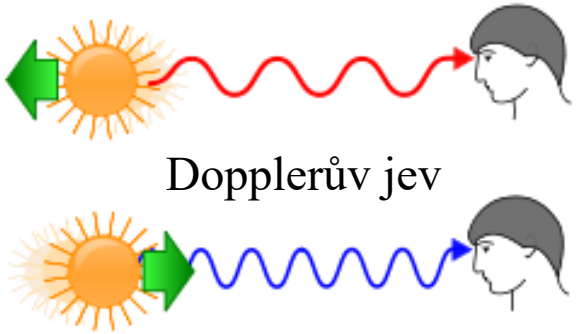

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Teorie gravitace

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Ricciho tenzor — Skalární křivost — Metrický tenzor — Kosmologická konstanta — Tenzor energie-hybnosti

Expanze Vesmíru

$$H^2 = \left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{\Lambda}{3} - \frac{kc^2}{R^2(t)} + \frac{8\pi G}{3}\rho$$

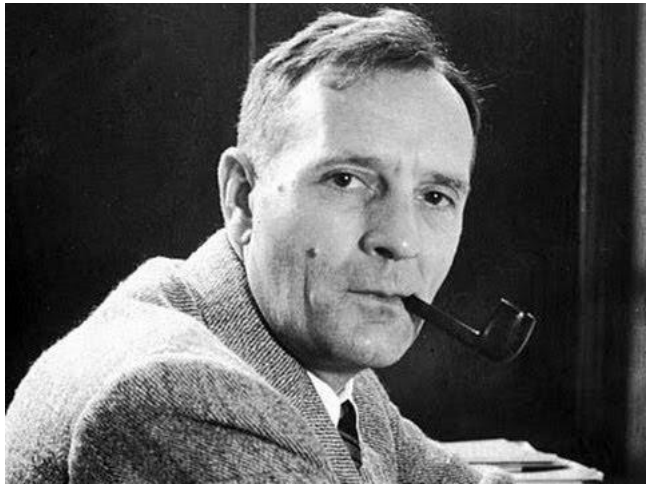
Parametr expanze — Kosmologická konstanta — Křivost prostoru — Hustota látky ~$R^{-3}(t)$ — Hustota záření ~$R^{-4}(t)$

$$\Omega_{total} = \frac{\rho}{\rho_{krit}}$$

$$\Omega_{total} = \Omega_{\Lambda} + \Omega_{mass} + \Omega_{relativistic}$$

Rozpínající Vesmír

Friedmann–Lemaître–Robertson–Walker (**FLRW modely**)



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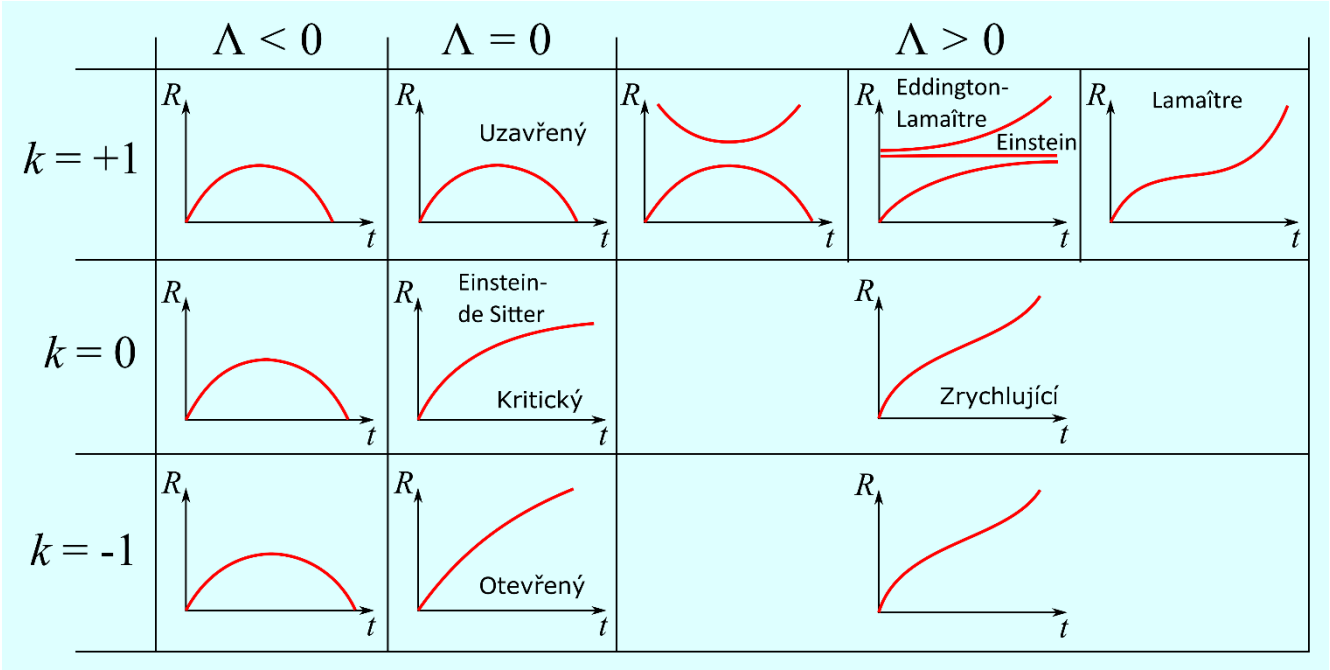

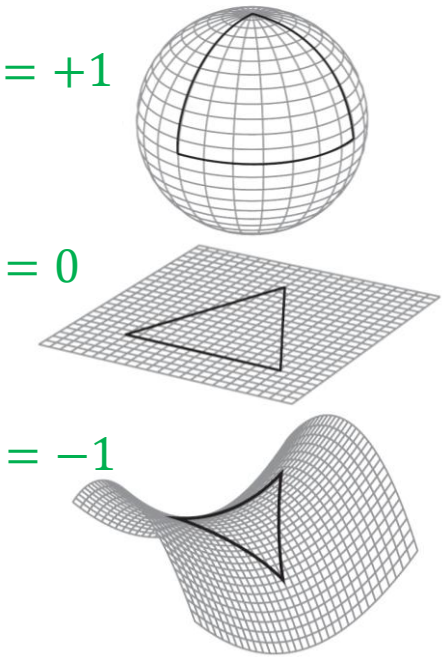

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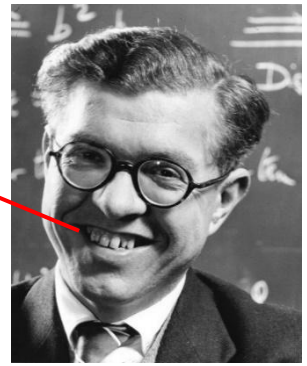
αβγ teorie – Teorie horkého Vesmíru (1949) „Teorie **Big Bang**“

G. Gamow R. A. Alpher R. C. Herman



F. Hoyle

- Vysvětlení vzniku lehkých prvků (H~80%, He ~20%, Li, Be)

- Oddělení záření od hmoty – **Reliktní záření**
  - cca 400 000 let (teplota 5 000 K, E = 0,4 eV)
  - Dnes teplotu cca 5 K => mikrovlnné záření
  - Vyplňuje celý Vesmír přibližně homogenně

The Origin of Chemical Elements

R. A. Alpher*
Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland

AND

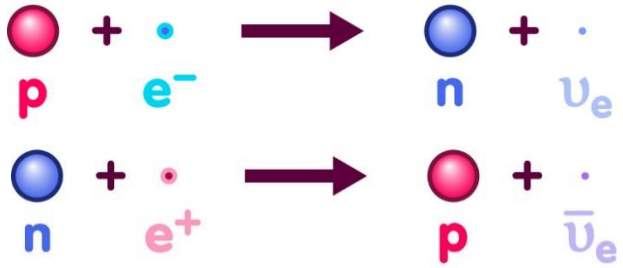
H. Bethe
Cornell University, Ithaca, New York

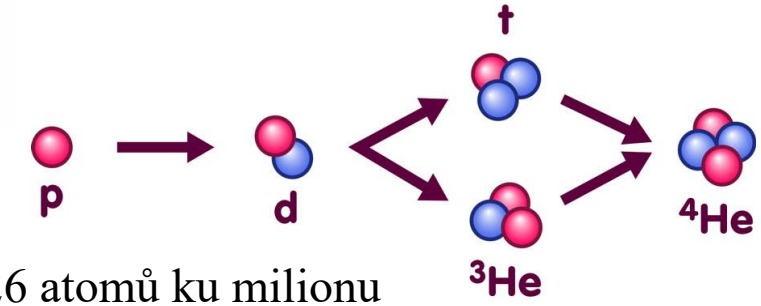
AND

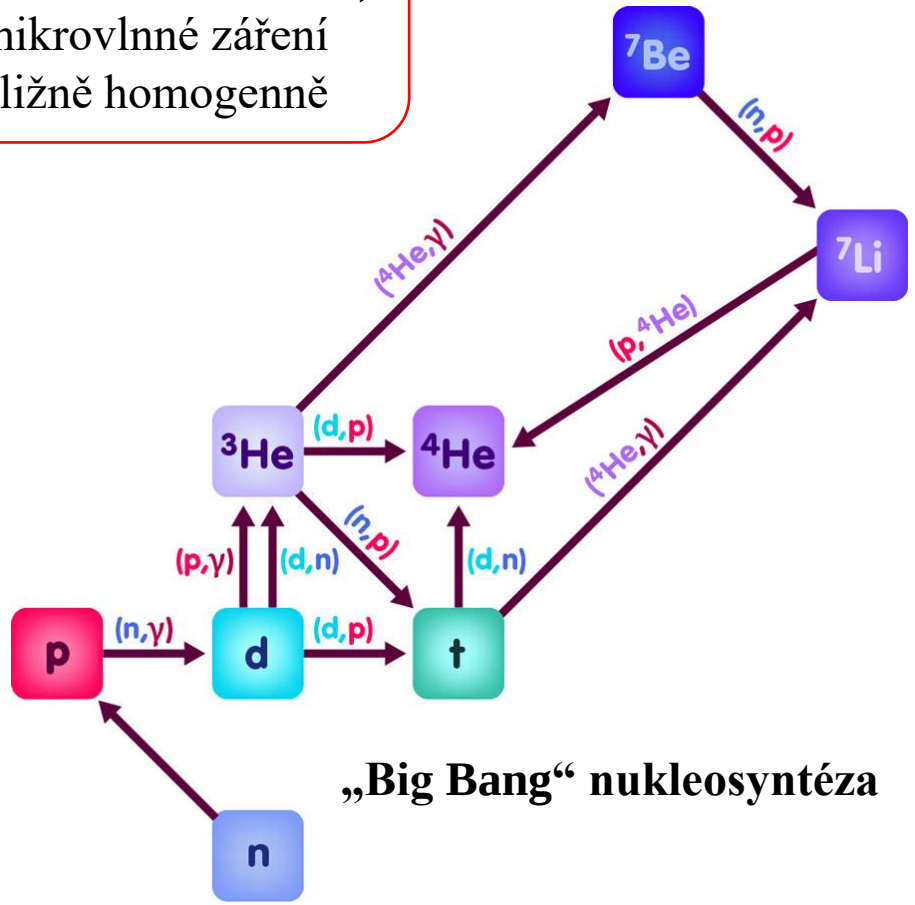
G. Gamow
The George Washington University, Washington, D. C.

February 18, 1948

Beta rozpad



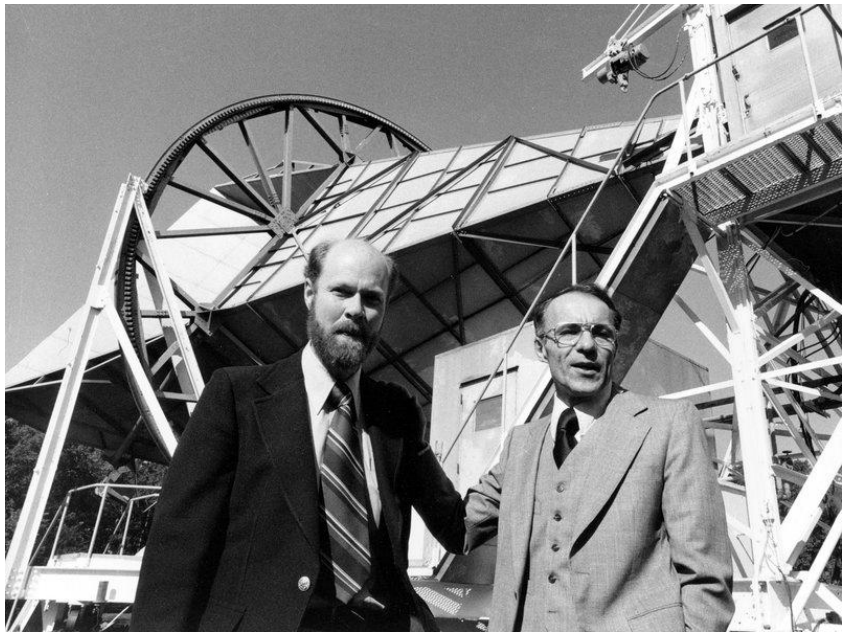




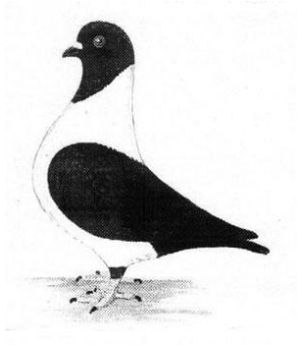
„Big Bang“ nukleosyntéza

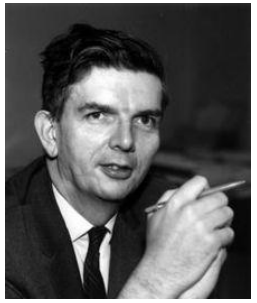
Poměr deuterium ku vodíku (=protium): 26 atomů ku milionu

Objev Reliktního záření



A. A. Penzias
R. W. Wilson





R. H. Dicke

J. Peebles

THE ASTROPHYSICAL JOURNAL

We deeply appreciate the helpfulness of Drs. Penzias and Wilson of the Bell Telephone Laboratories, Crawford Hill, Holmdel, New Jersey, in discussing with us the result of their measurements and in showing us their receiving system. We are also grateful for several helpful suggestions of Professor J. A. Wheeler.

R. H. Dicke
P. J. E. Peebles
P. G. Roll
D. T. Wilkinson

May 7, 1965
Palmer Physical Laboratory
Princeton, New Jersey

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Wheeler, J. A , 1958, *La Structure et l'évolution de l'universe* (11th Solvay Conf. [Brussels: Éditions Stoops]), p. 112.
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el'dovich, Ya. B. 1962, *Soviet Phys.—J.E.T.P.*, **14**, 1143.

A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE AT 4080 Mc/s

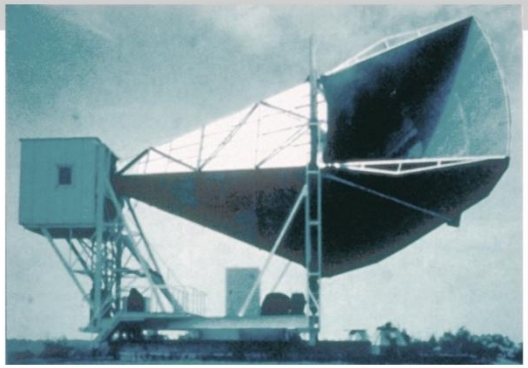
Measurements of the effective zenith noise temperature of the 20-foot horn-reflector antenna (Crawford, Hogg, and Hunt 1961) at the Crawford Hill Laboratory, Holmdel, New Jersey, at 4080 Mc/s have yielded a value about 3.5° K higher than expected. This excess temperature is, within the limits of our observations, isotropic, unpolarized, and

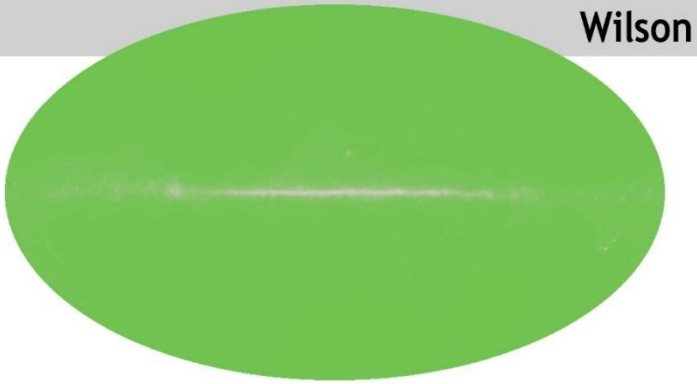
A. A. Penzias
R. W. Wilson

May 13, 1965
Bell Telephone Laboratories, Inc
Crawford Hill, Holmdel, New Jersey

1965

Penzias and Wilson



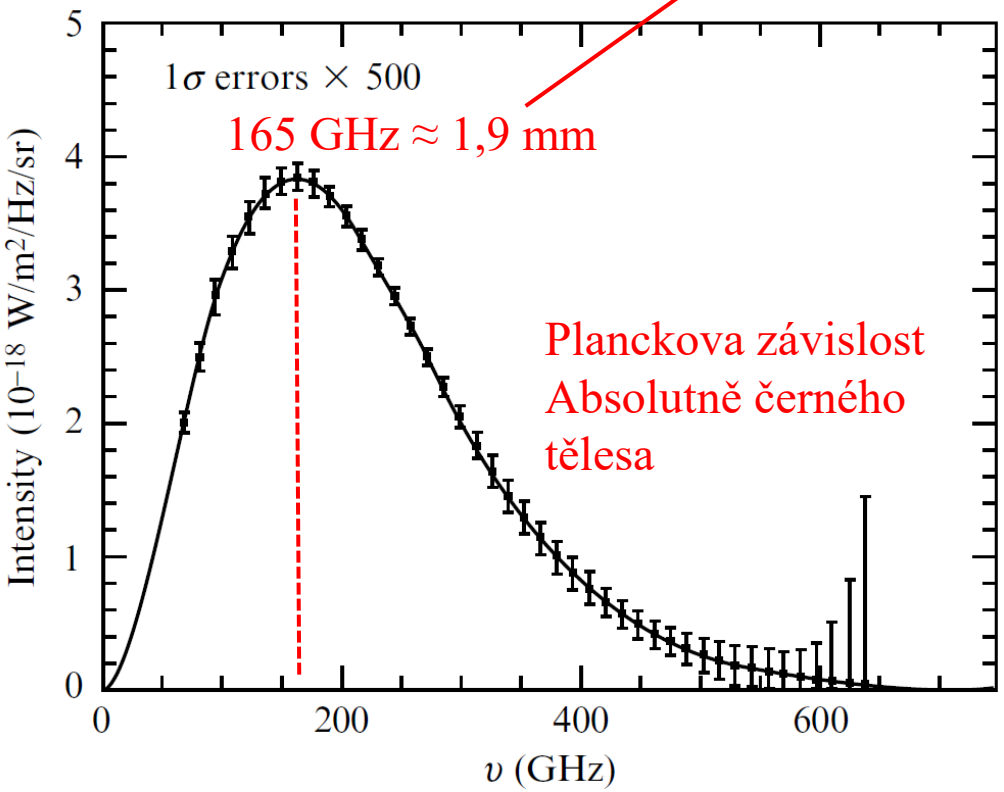


Teplota záření odpovídá ≈ 3,5 K

Výzkum Reliktního záření

- **COBE** (Cosmic Background Explorer)
- První návrh 1974, start rakety 1988
- Tři detektory mikrovlnného záření

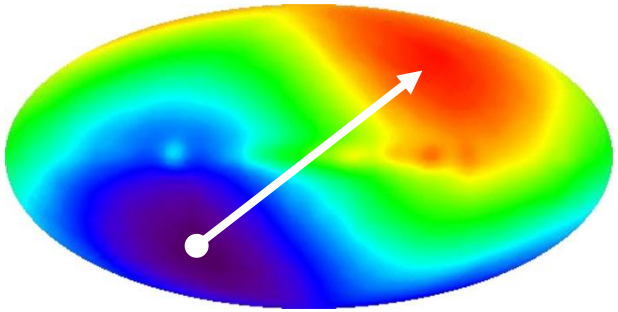
- 18. 11. 1989 změřena odpovídající teplota mikrovlnného reliktního záření **2,725 K**

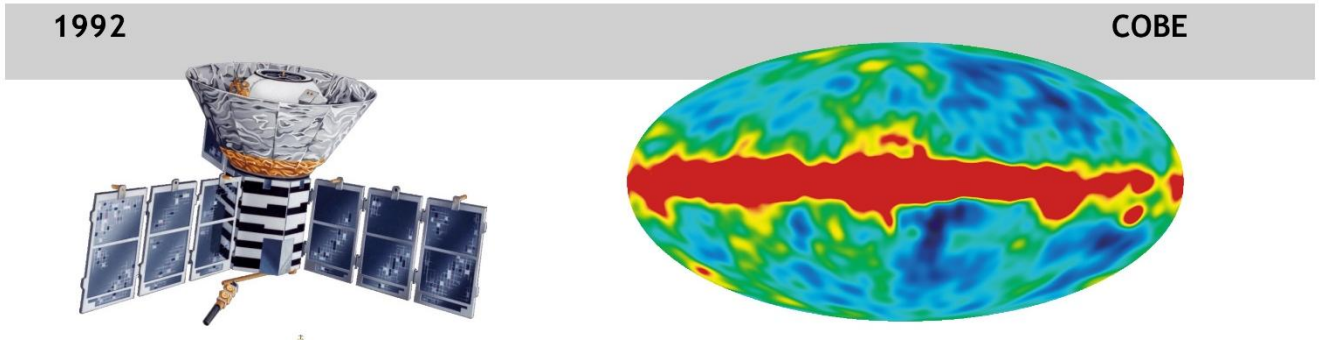


Dopplerův jev 371 km/s

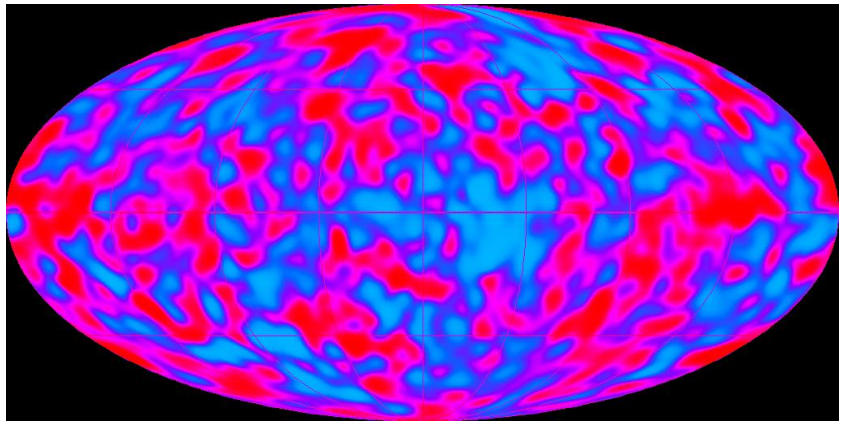
1976 letadlo U-2

Dipólová anizotropie
$\Delta T \approx 3$ mK





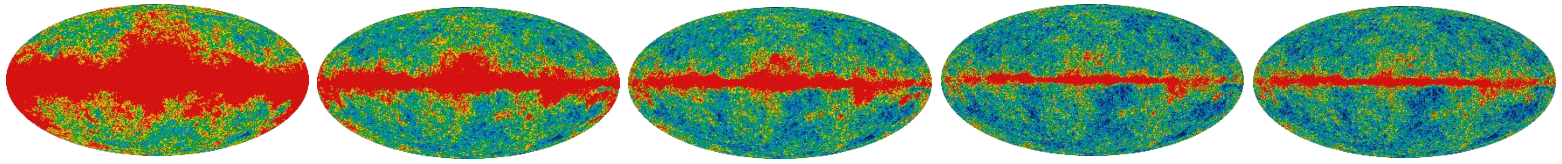
23. dubna 1992 Objev fluktuací teploty => **anizotropie reliktního záření**

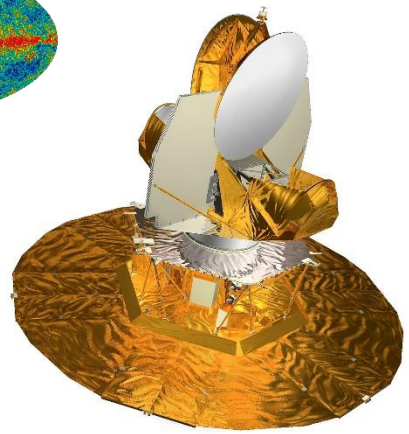


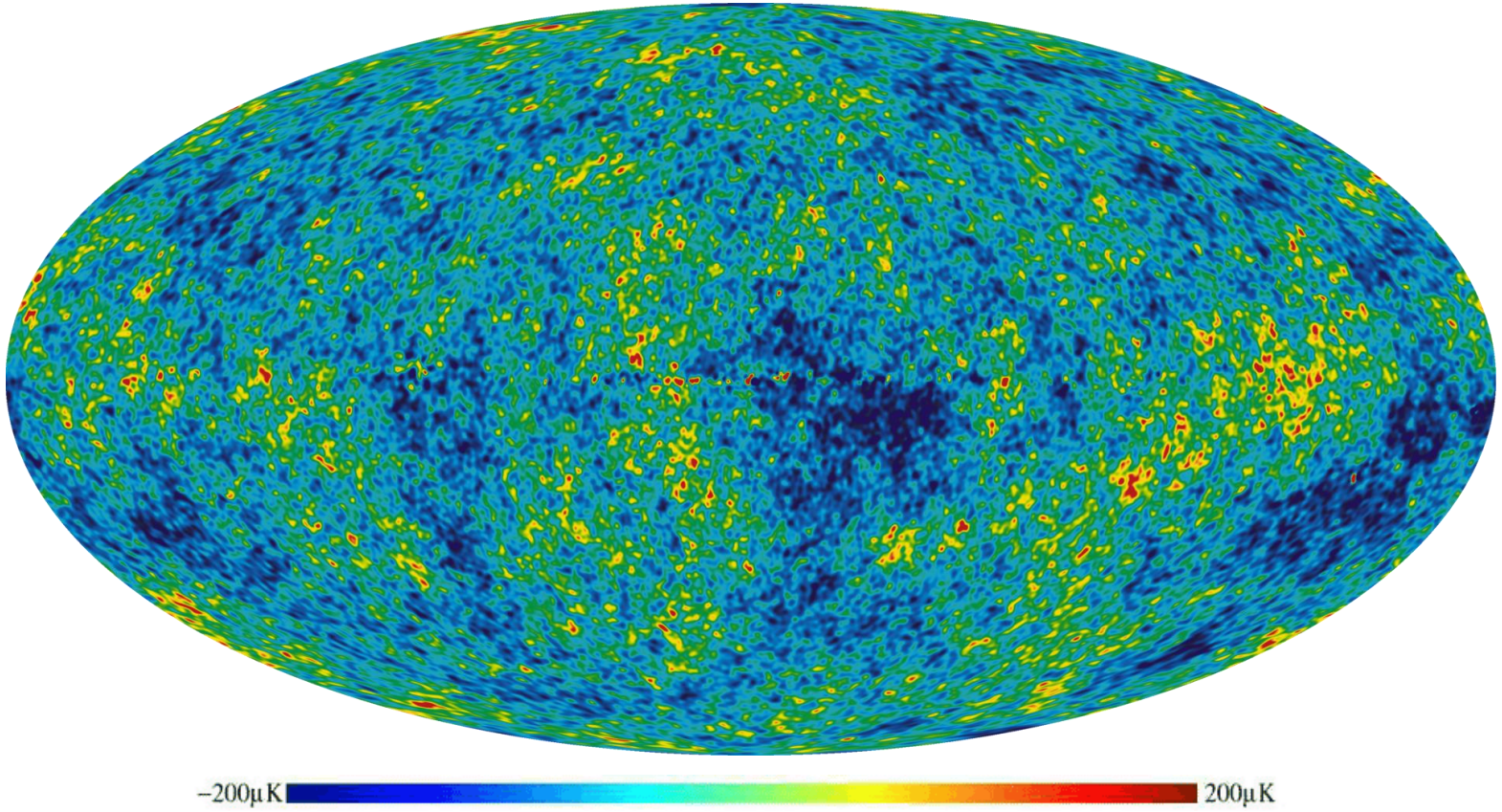
$\Delta T \approx 10^{-5}$ K

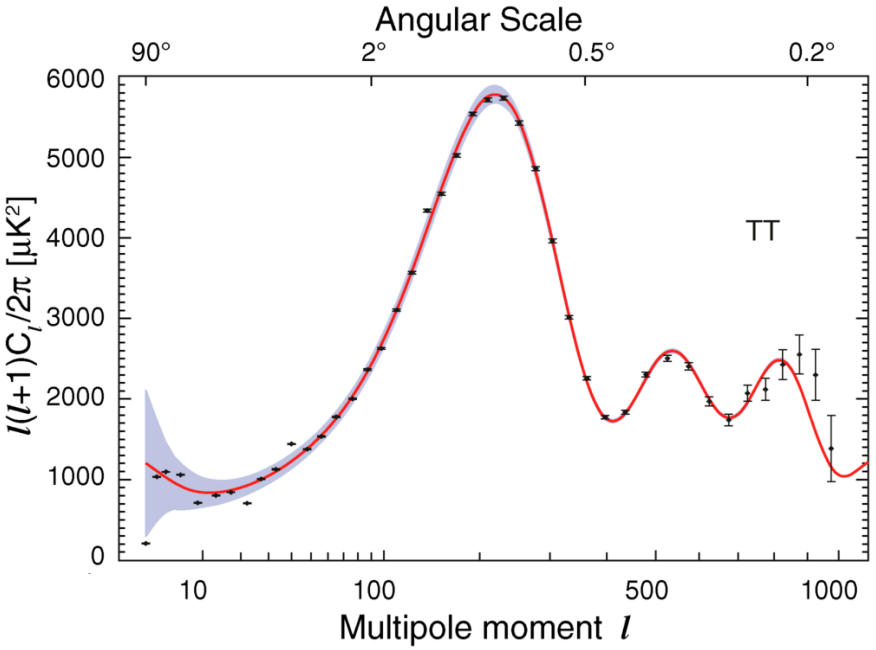
WMAP (Wilkinson Microwave Anisotropy Probe)

- Start 2001
- První výsledky 2003
- (2006, 2008, 2010, 2012)
- 5 detektorů

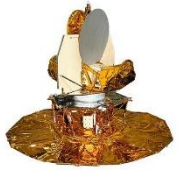


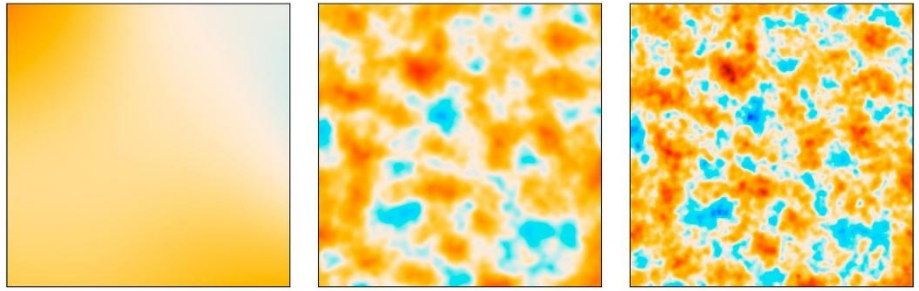


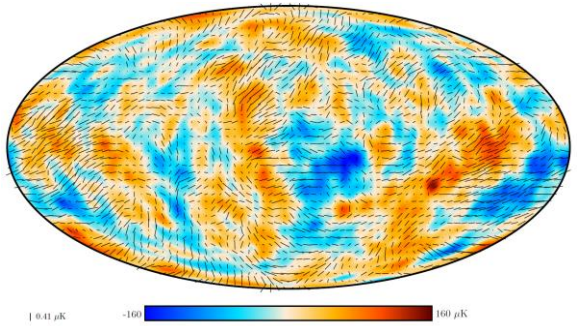


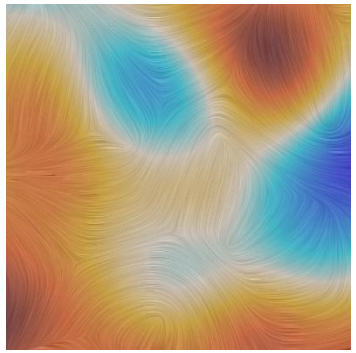


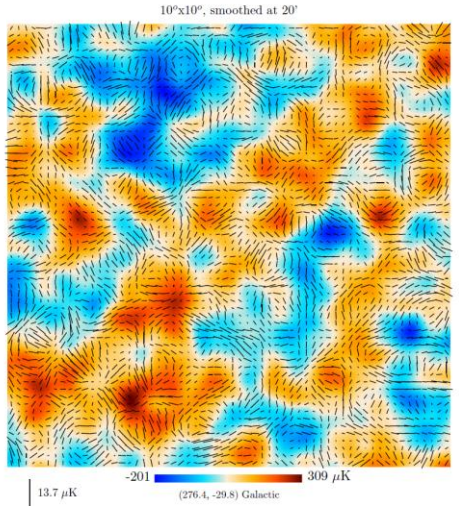
Sonda Planck



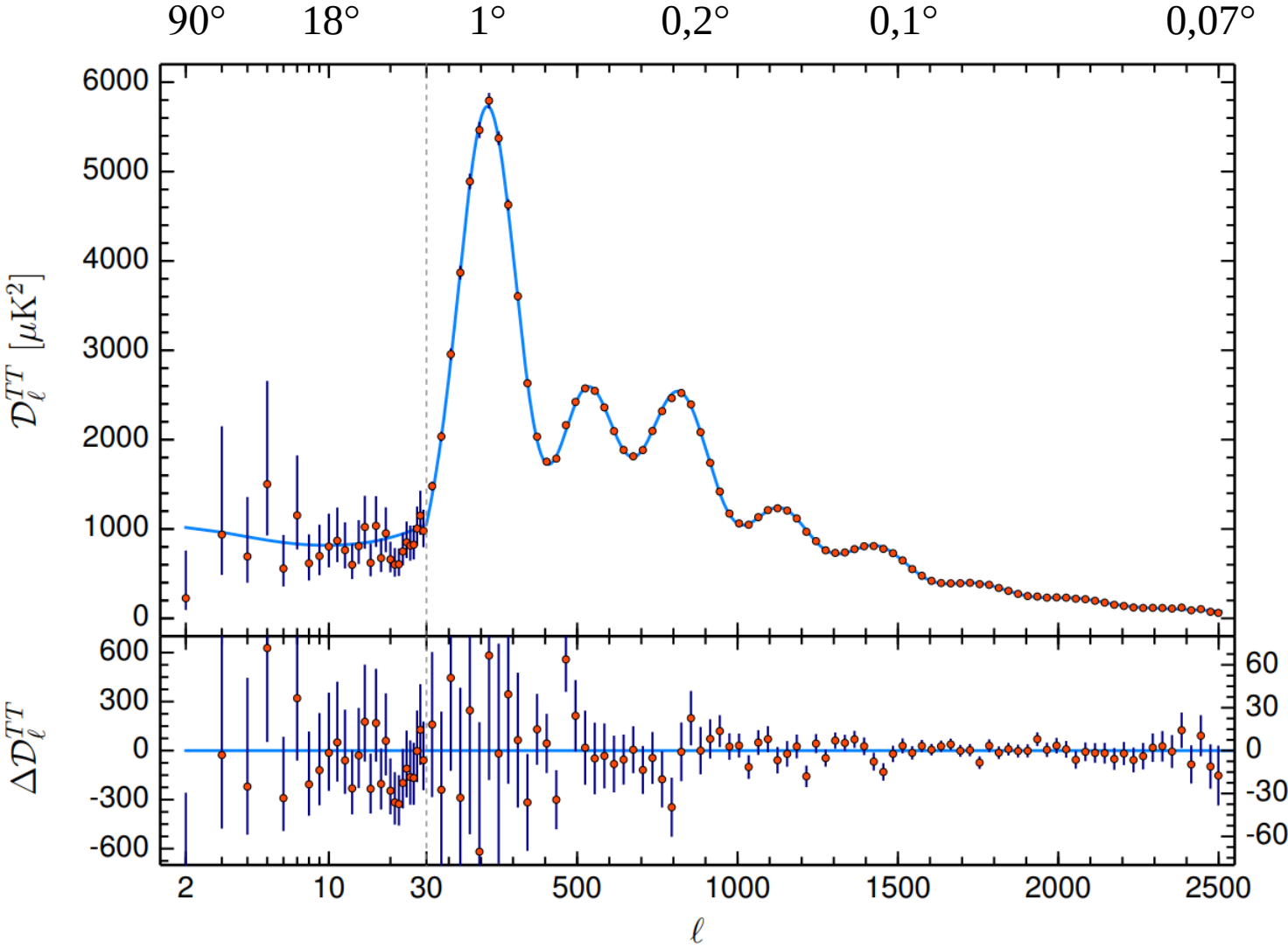




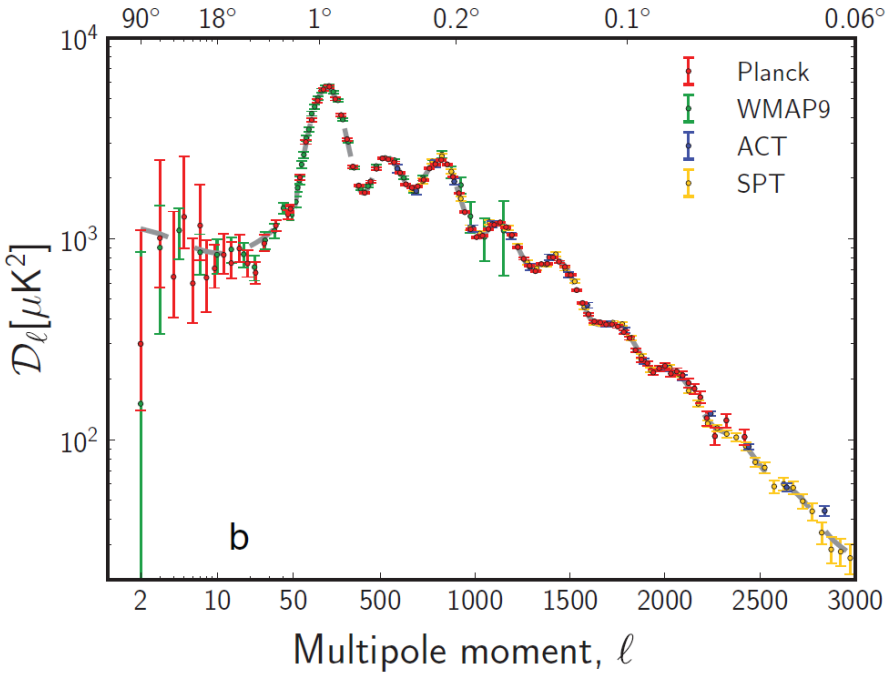


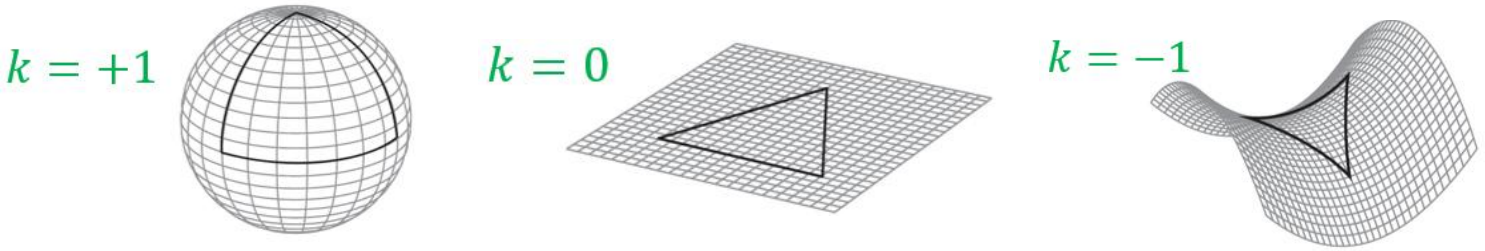


- Start 2009
- První výsledky 2013 (2015, 2018 zpřesnění)
- 9 detektorů (30 – 857 GHz) + polarizační měření



Kosmologické parametry





Staří Vesmíru	$13{,}787 \times 10^9$ ($\pm\ 0{,}020 \times 10^9$)	let
Oddělení reliktního záření	377 700 ± 3200	let
Rudý posuv reliktního záření	$z = 1089{,}9 \pm 0{,}23$	
Kritická hustota	$8{,}62 \times 10^{-27}$ ($\pm\ 0{,}12 \times 10^{-28}$)	$kg \cdot m^{-3}$
Poměr celkové hustoty ku kritické	1,02 (± 0,02)	
Křivost Vesmíru	Velmi plochý ($k \approx 0$)	
Kosmologická konstanta	$\Lambda = +1{,}1 \times 10^{-52}$	m^{-2}
Hubbleova konstanta	$H_0 = 67{,}4 \pm 0{,}5$	$km \cdot s^{-1} \cdot Mpc^{-1}$

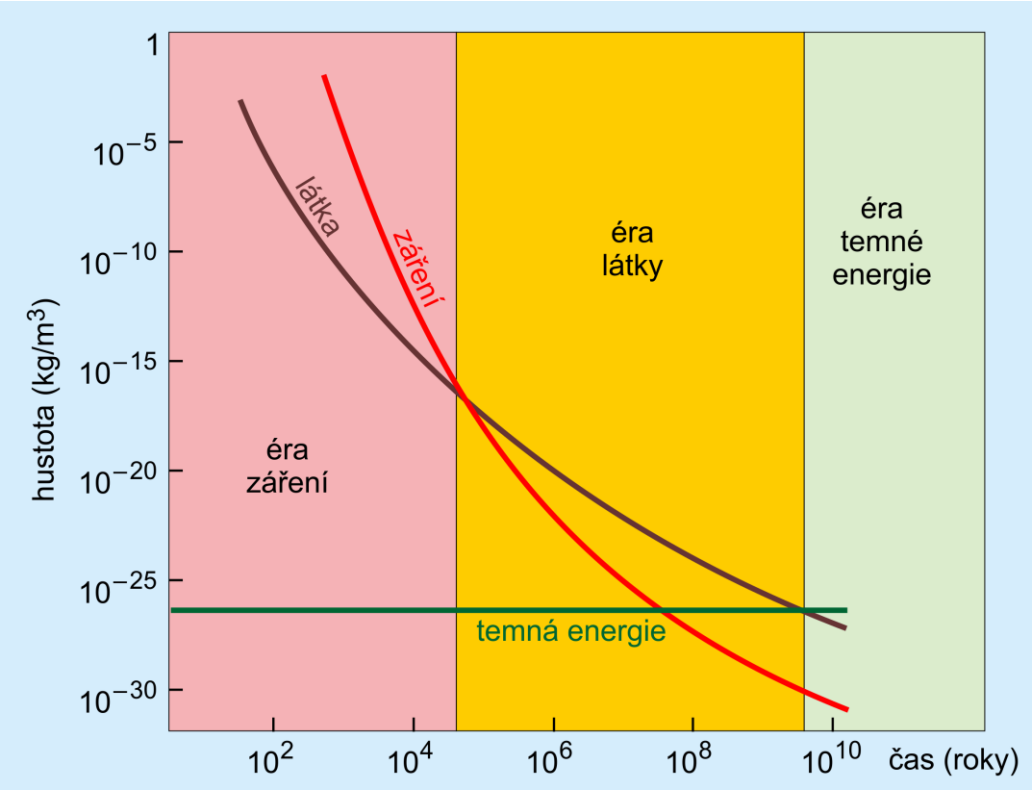
$$\Omega_{total} = \Omega_\Lambda + \Omega_{mass} + \Omega_{relativistic}$$

$$\Omega_{total} = \frac{\rho}{\rho_{crit}} \qquad \rho_{crit}(t) = \frac{3H^2(t)}{8\pi G}$$

$$H(t) = \frac{\dot{R}(t)}{R(t)} = H_0 \sqrt{\frac{\Omega_{mass}}{R^3(t)} + \frac{\Omega_{rel}}{R^4(t)} + \Omega_\Lambda}$$

$$H^2 = \left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{\Lambda}{3} - \frac{kc^2}{R^2(t)} + \frac{8\pi G}{3}\rho$$

Λ-CDM model Vesmíru



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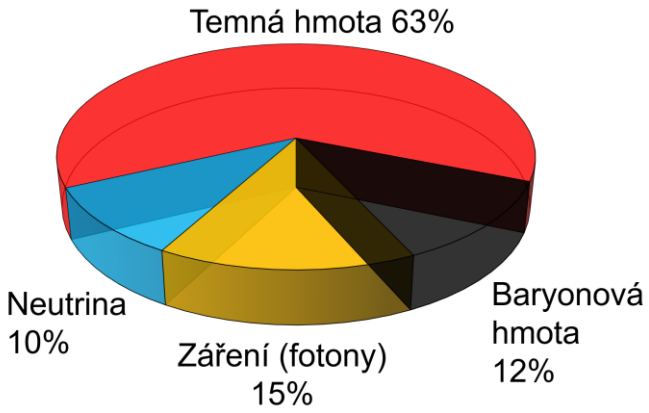
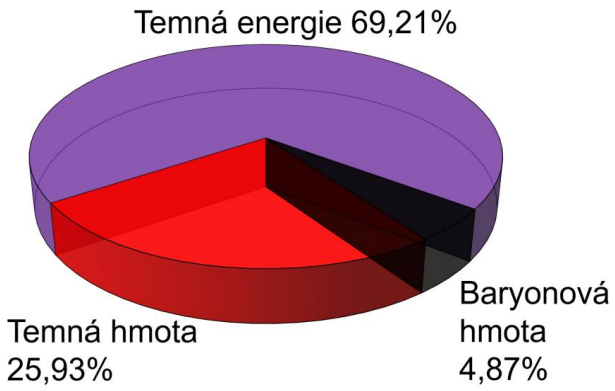
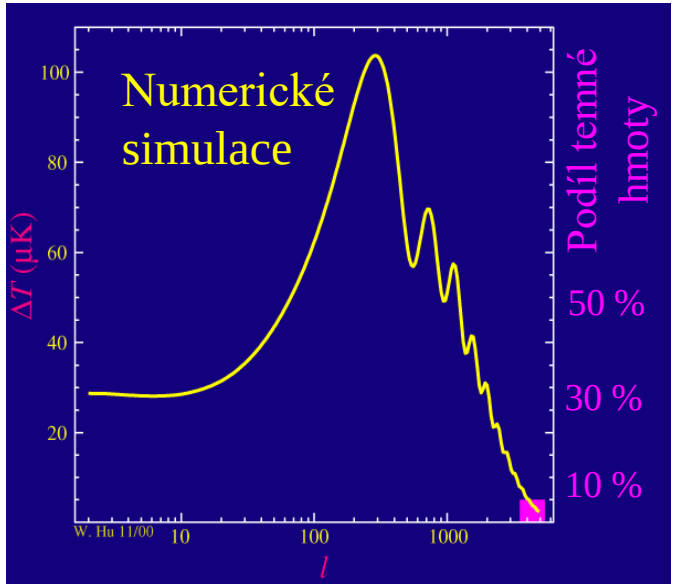
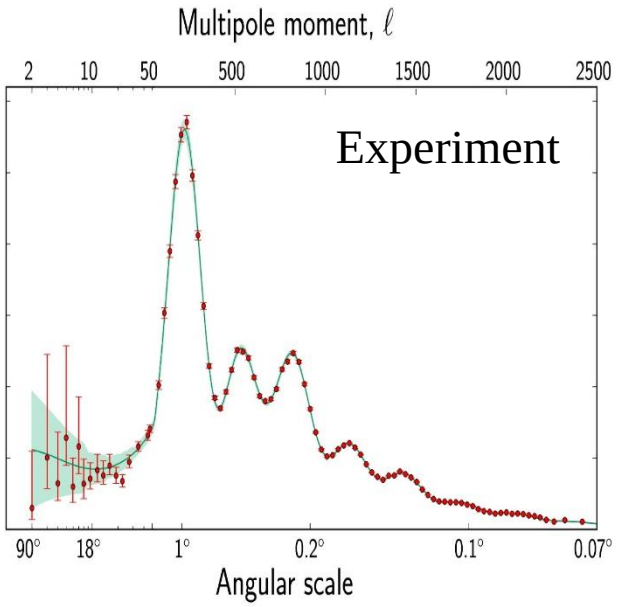
(Před 13,7 mld. let)



(Dnes)

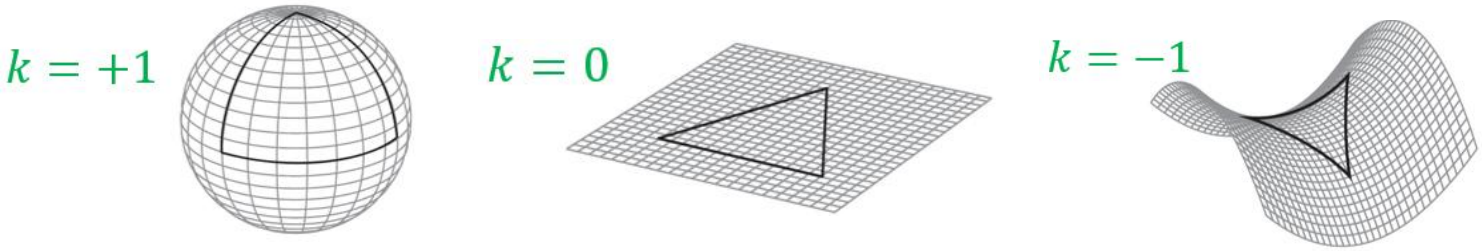




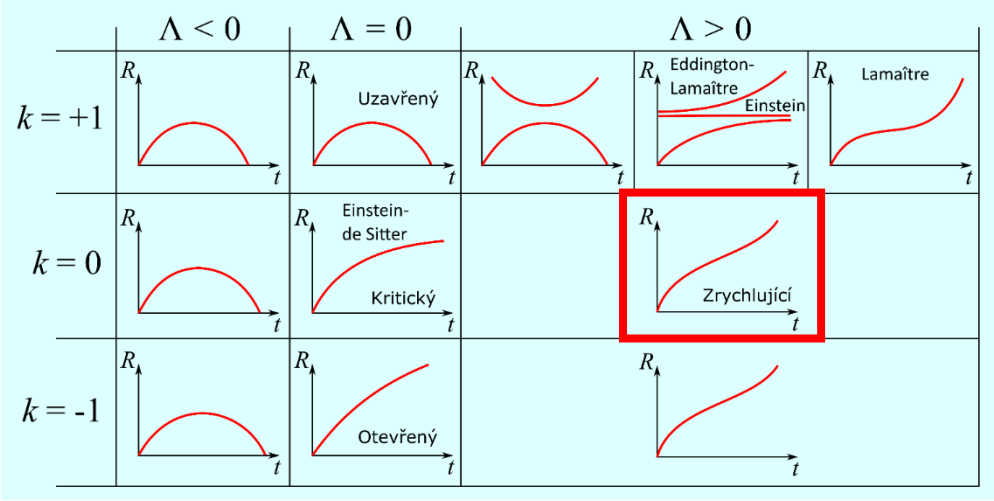


http://background.uchicago.edu/~whu/intermediate/intermediate.html

Kosmologické parametry



FLRW modely



=> Exponenciálně zrychlující Vesmír!

Ale kdy začne/začal zrychlovat?

Jak je dnes Vesmír velký?

Staří Vesmíru	$13{,}787 \times 10^9$ ($\pm\ 0{,}020 \times 10^9$)	let
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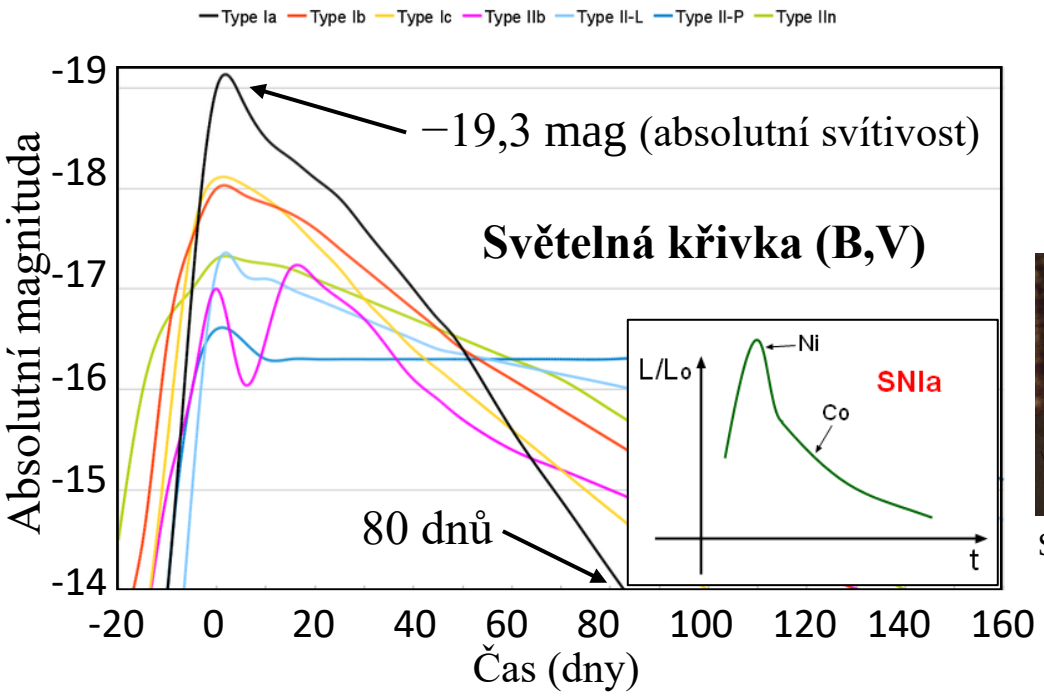
$$H^2 = \left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{\Lambda}{3} - \frac{kc^2}{R^2(t)} + \frac{8\pi G}{3}\rho$$

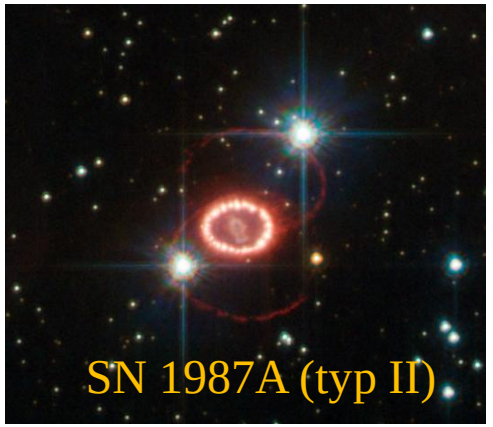
Zrychlující Vesmír

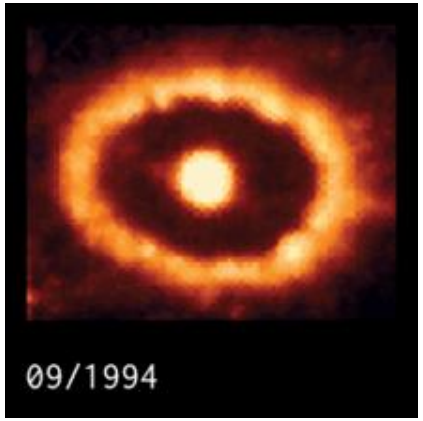


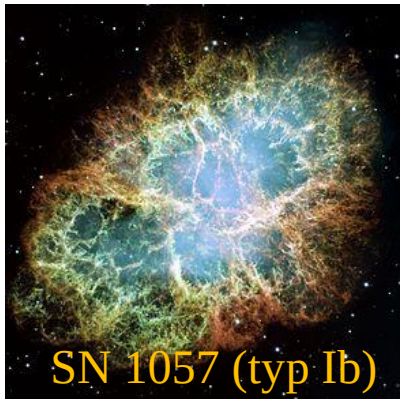
Nobelova cena 2011

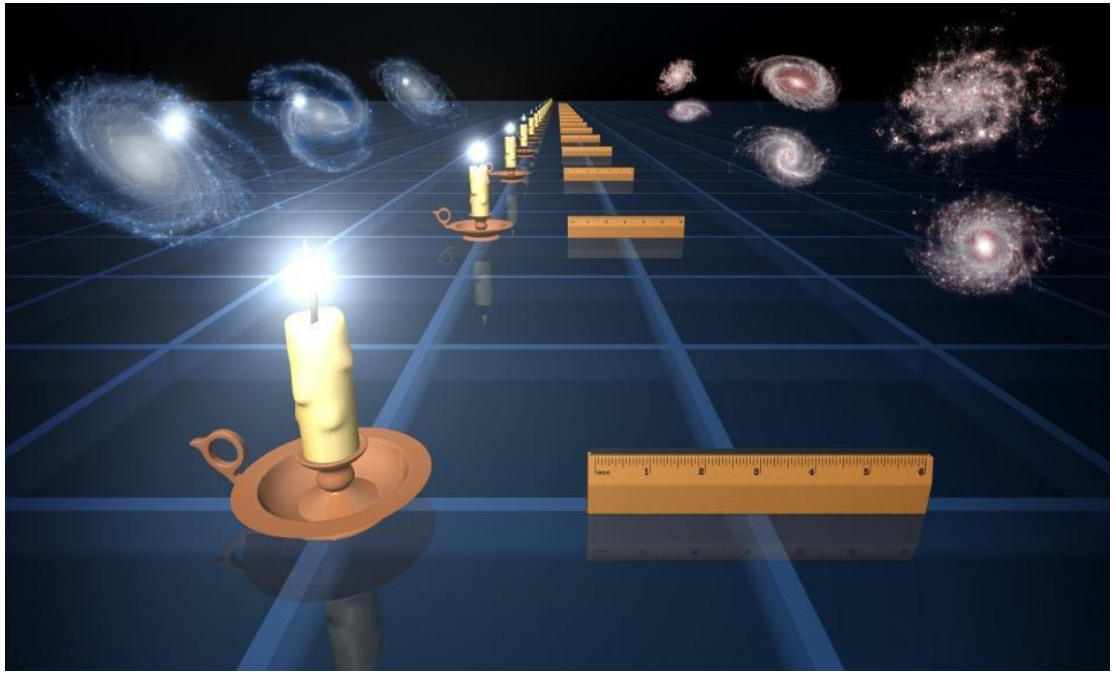
S. Perlmutter B. P. Schmidt A. G. Riess

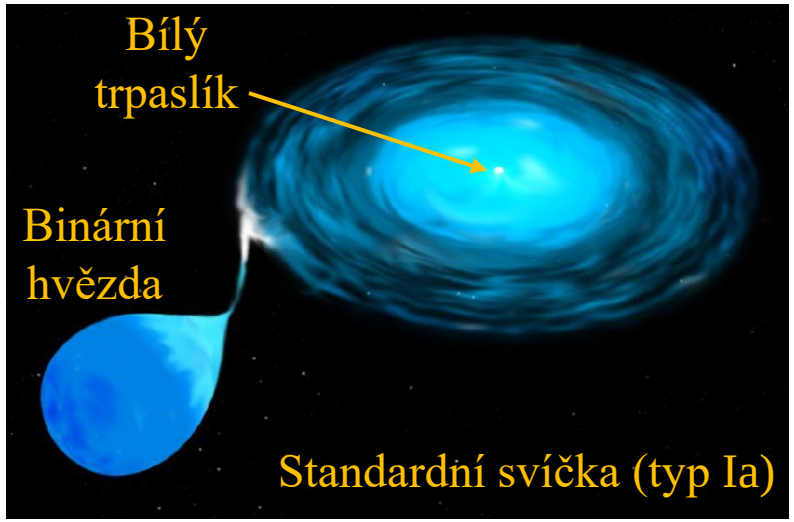
$$d = 10^{(m-M+5)/5}$$

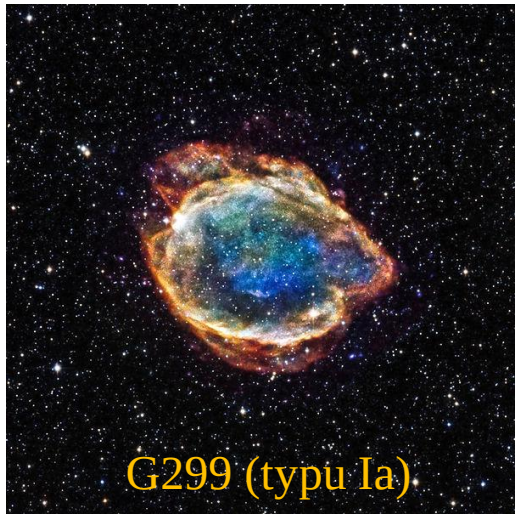




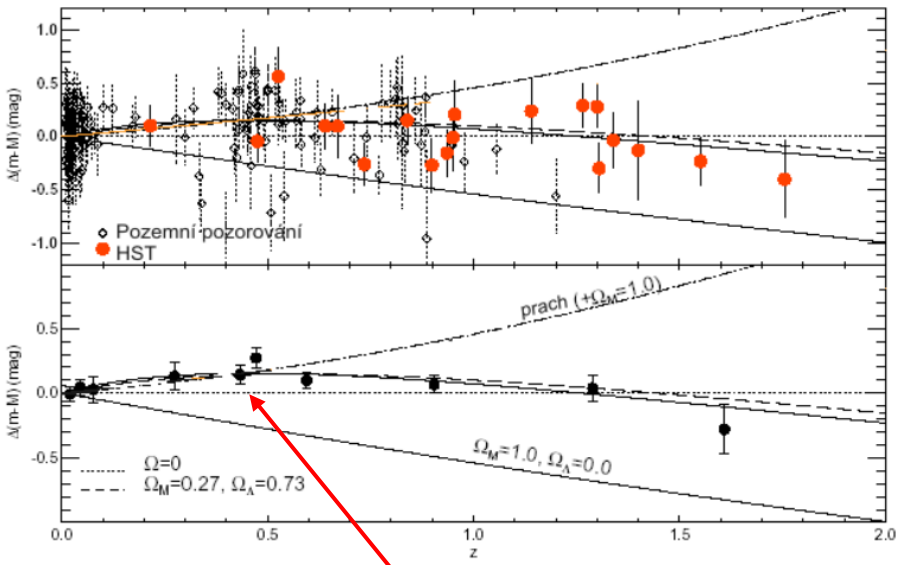






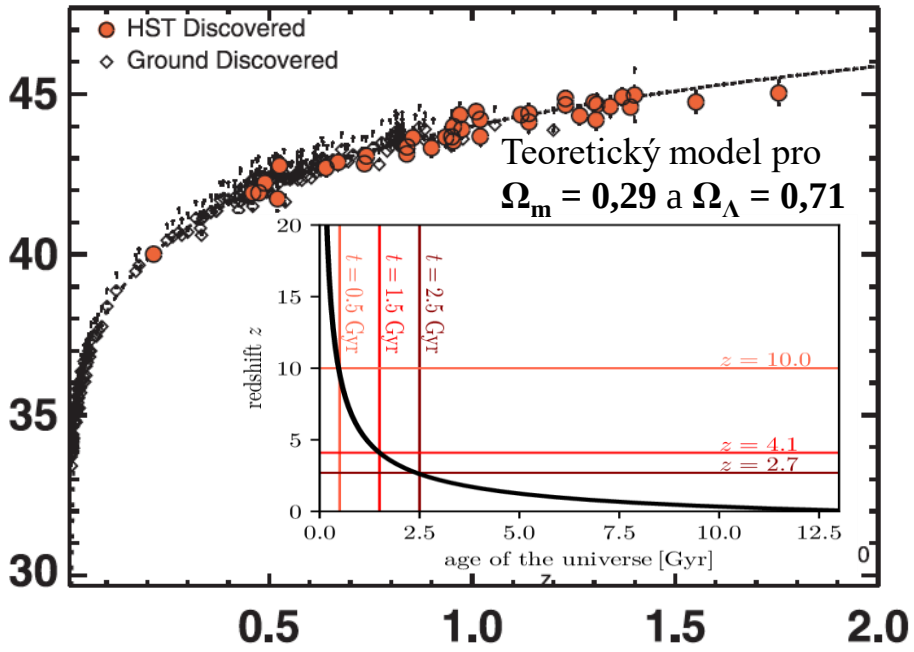


Zrychlující Vesmír



$z = 0{,}46 \pm 0{,}13$

stáří Vesmíru 8–9 mld. let

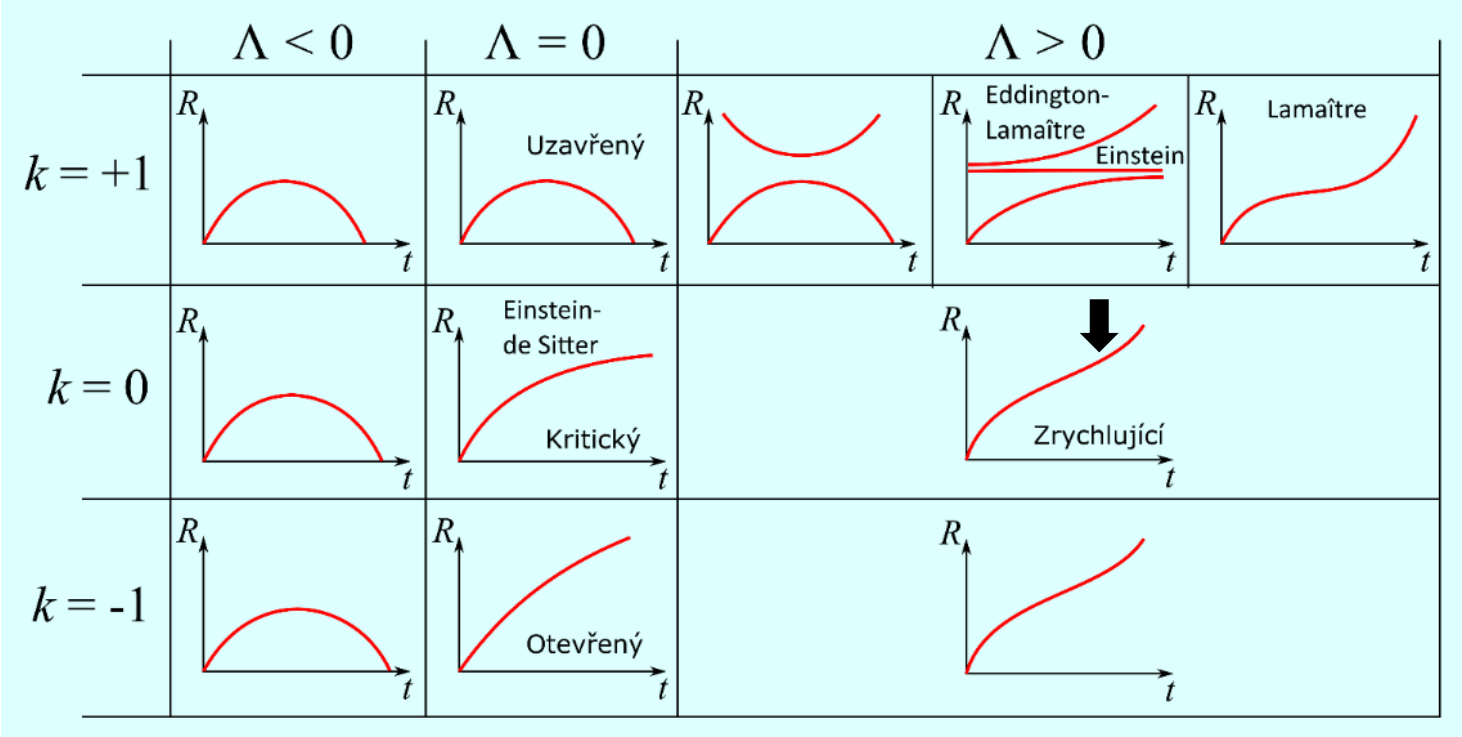


Rudý posuv z

Temná energie odpovídá kosmologické konstantě Λ

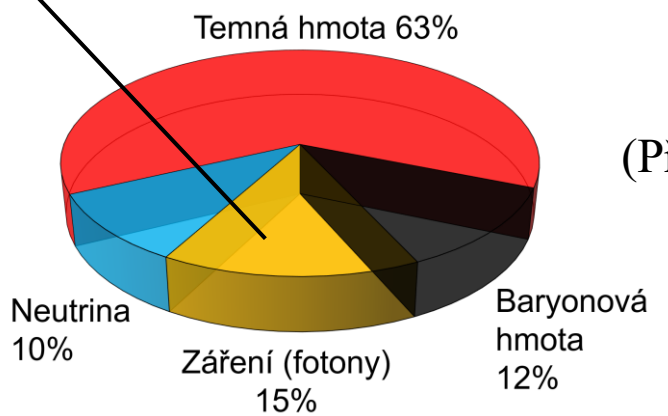
Nejdřív vládne gravitační interakce, pak záporný tlak

FLRW modely



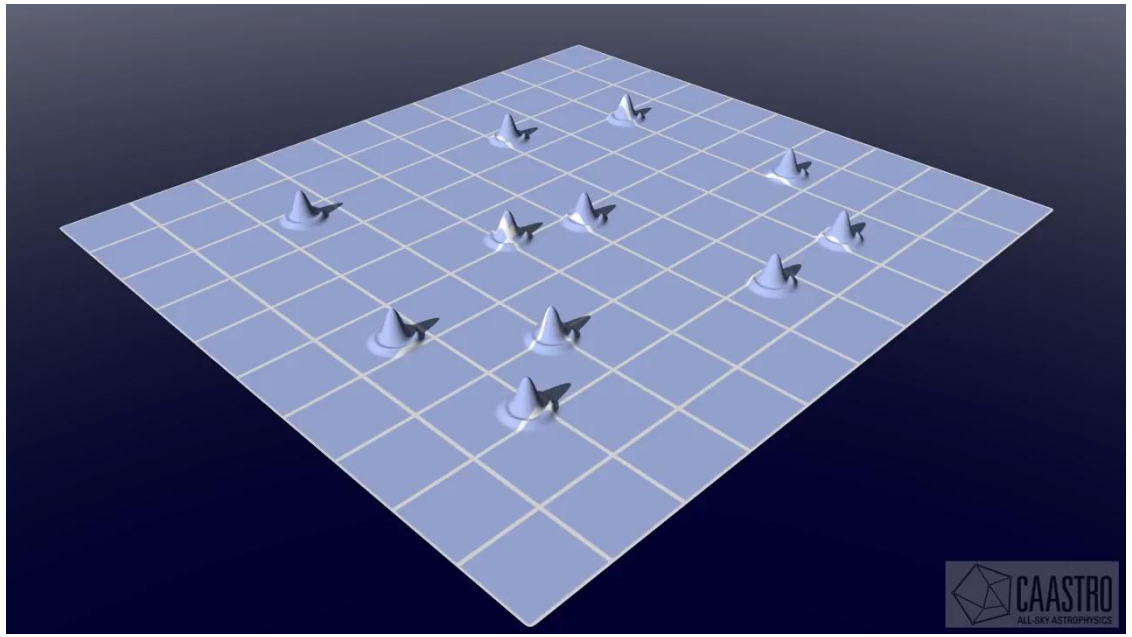
1) Zastoupení lehkých prvků ✓
2) Reliktní záření (CMB) ✓
3) Supernovy typu Ia (SNe) ✓
4) **Baryonové akustické oscilace (BAO)**

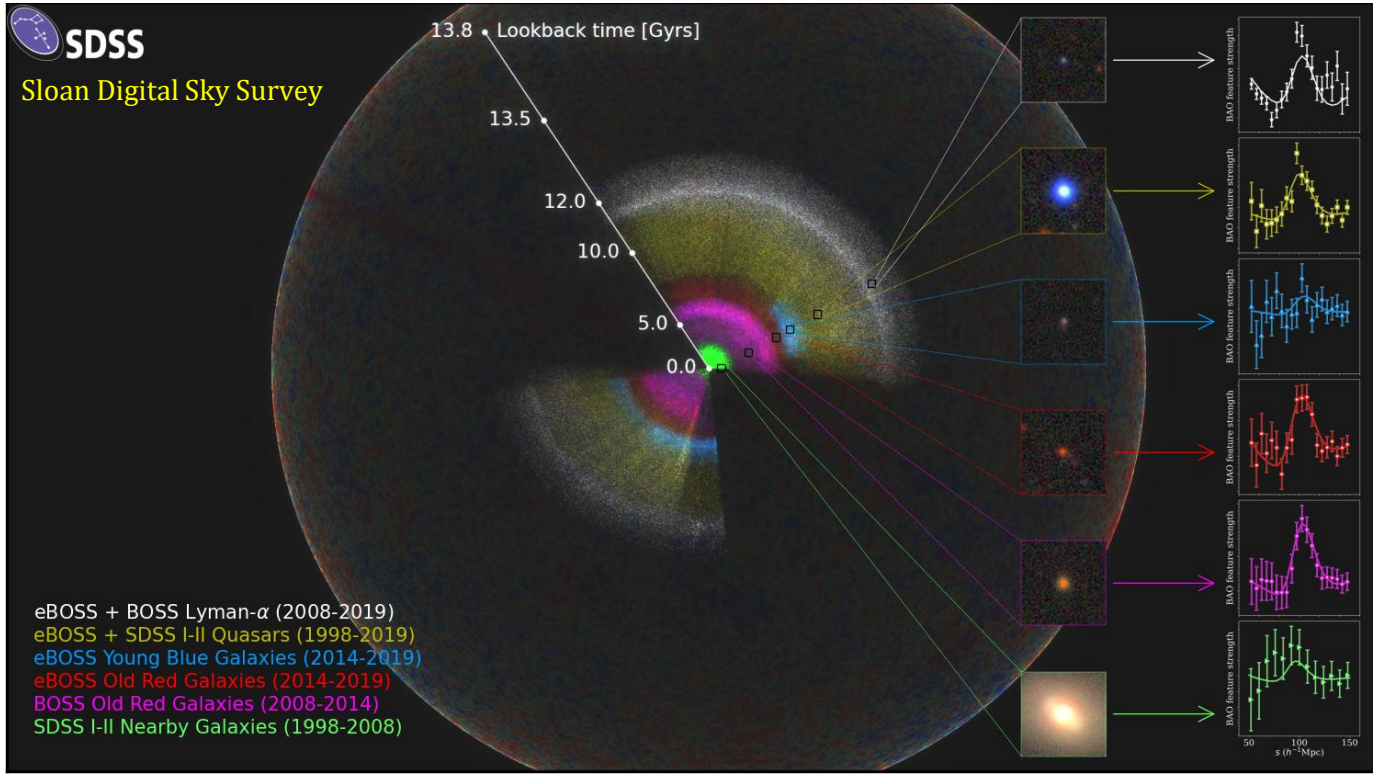
Fotony by rozmetaly baryonovou hmotu a nebyly by galaxie => **Temná hmota**



(Před 13,7 mld. let)

Vesmír stavěn **zdola → nahoru**
=> malé fluktuace v raném Vesmíru jsou zdrojem velkých struktur dnes





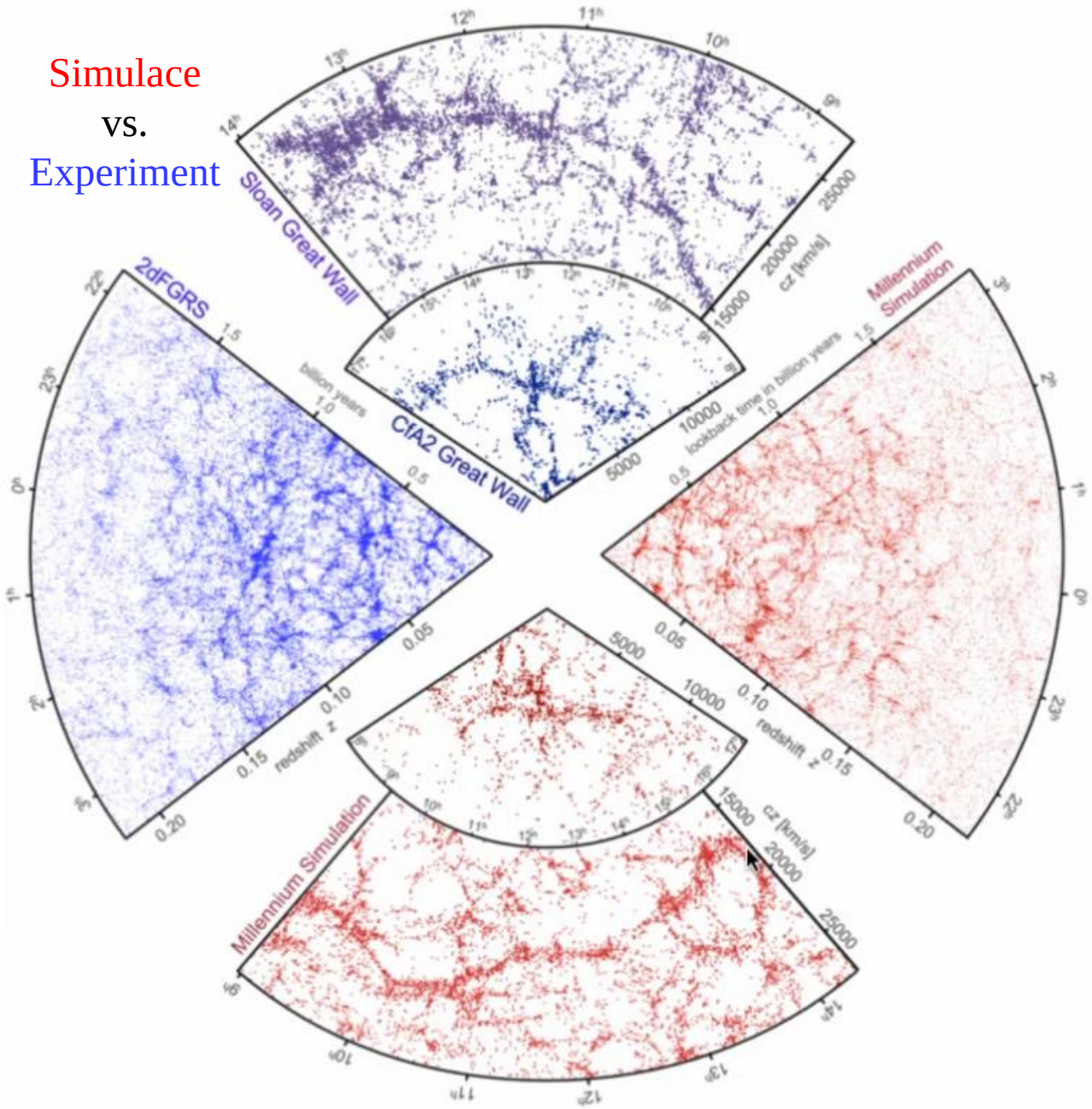
- Nerovnováha gravitačního tlaku vs. tlaku záření
  => zvukové oscilace plazmatu
- Plazma obsahuje baryonovou hmotu, temnou hmotu a fotony
- Temná hmota neinteraguje elektromagneticky, ale hlavně gravitačně
- BAO vznikají pouze do reliktního záření
- Dnes možné až 490 Mly velké struktury

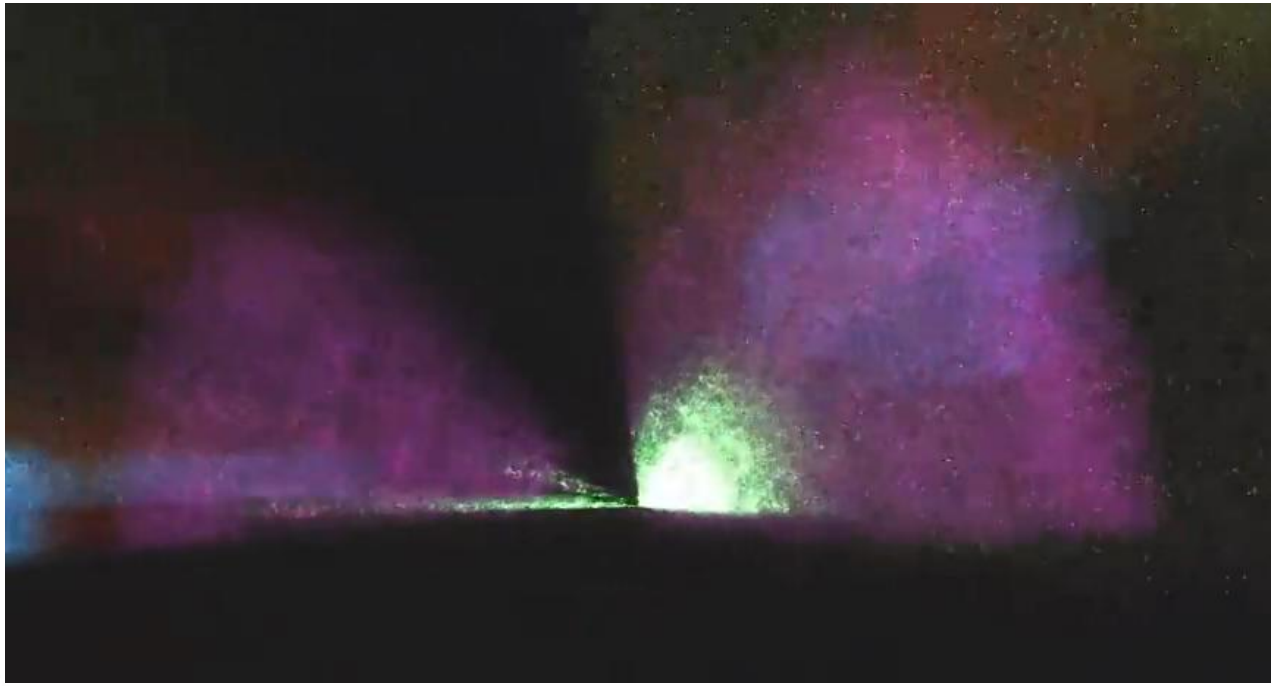
Struktura pozorovatelného Vesmíru

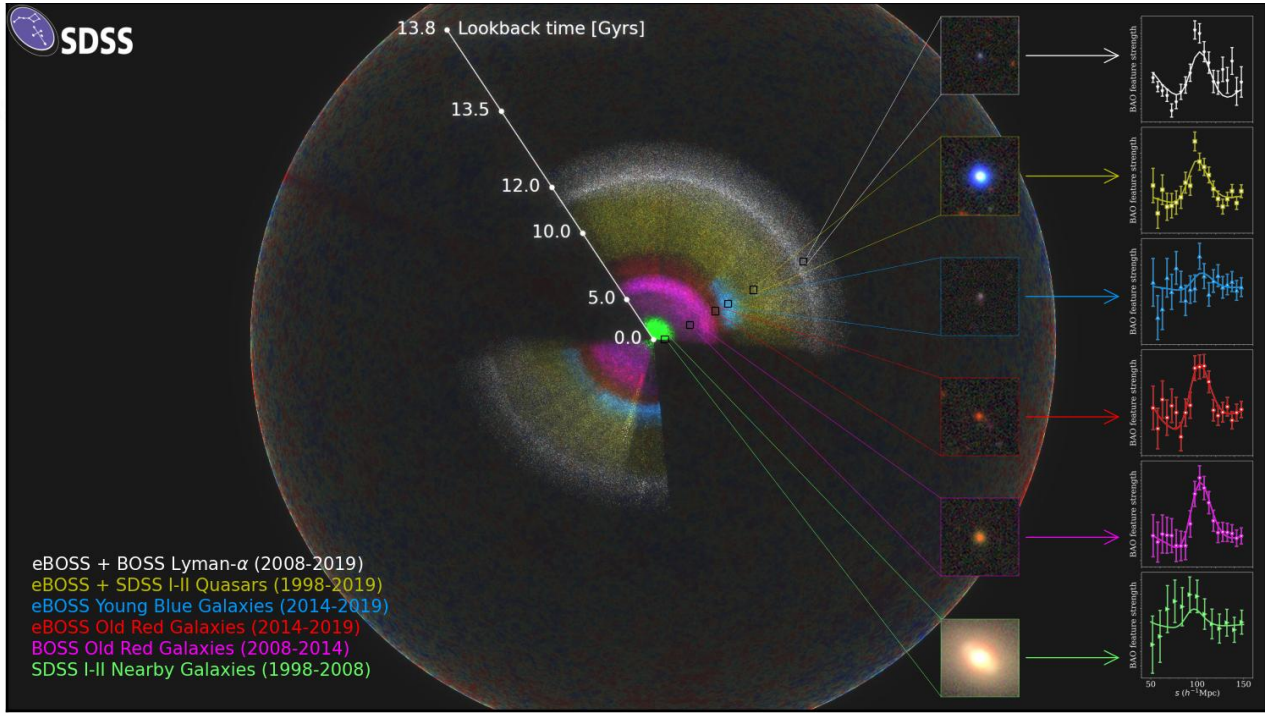
Simulace

vs.

Experiment





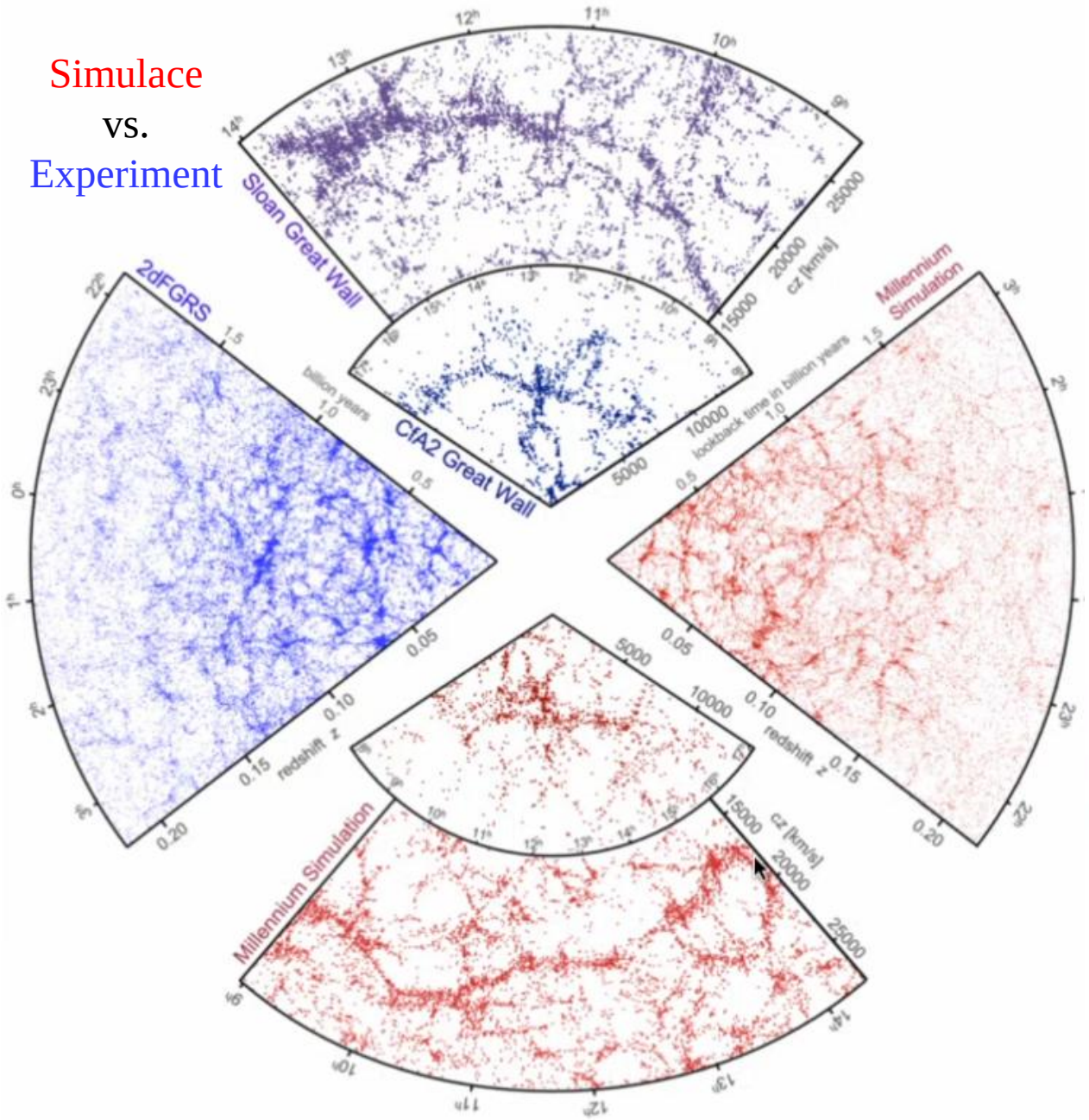


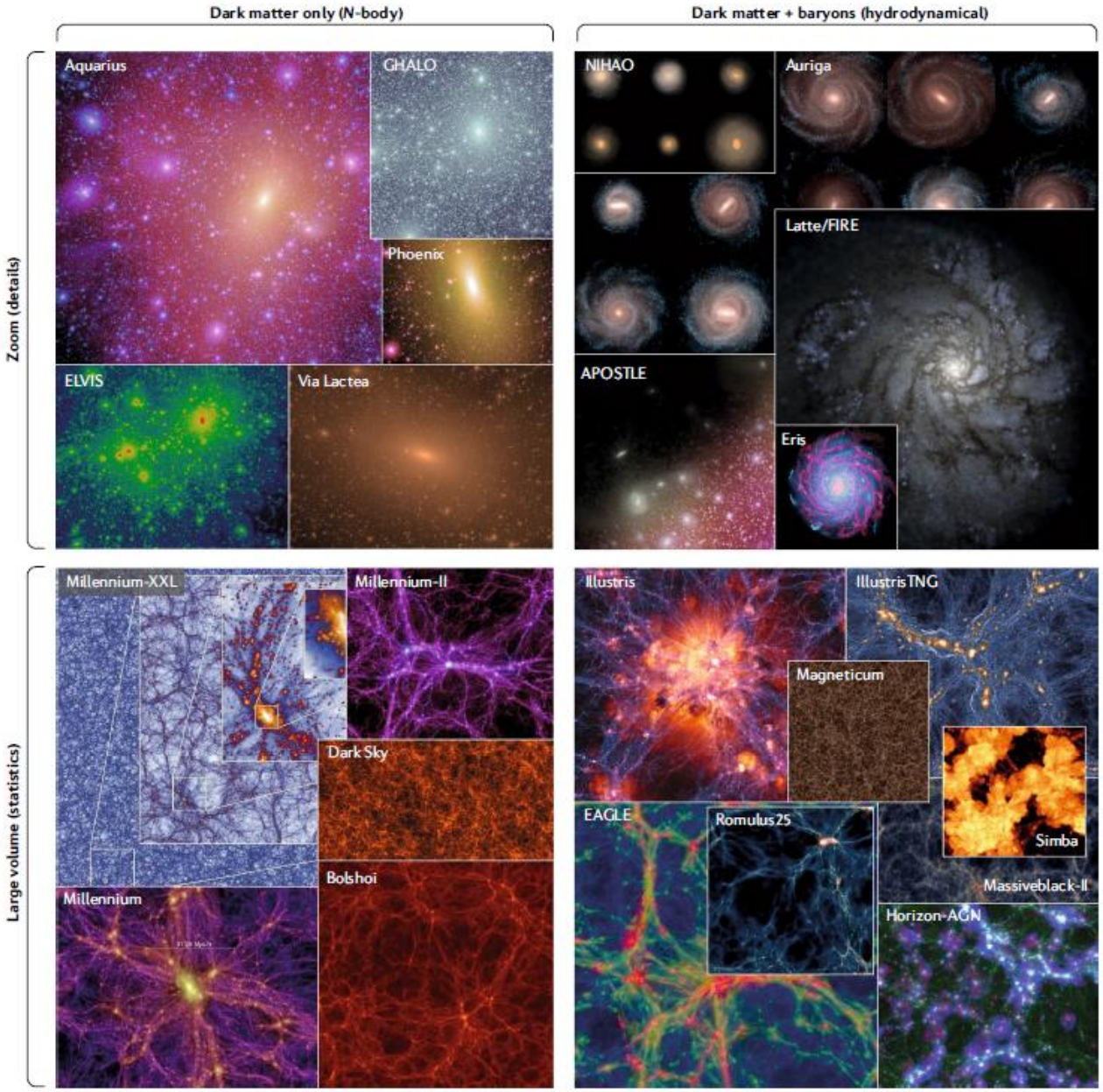
Struktura pozorovatelného Vesmíru

Simulace

vs.

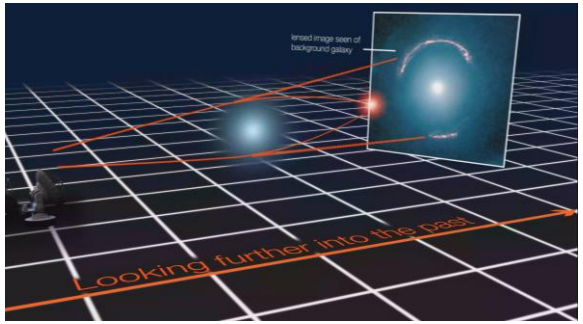
Experiment

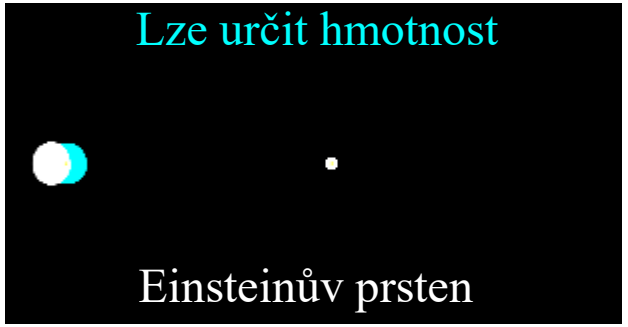


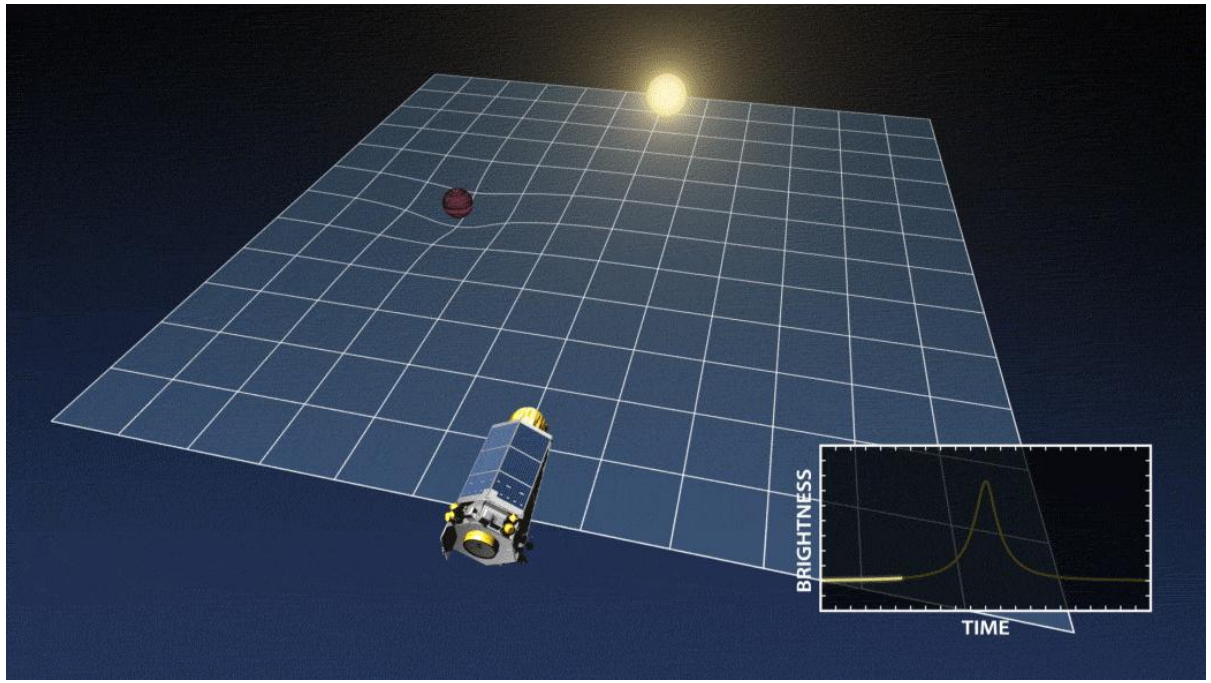


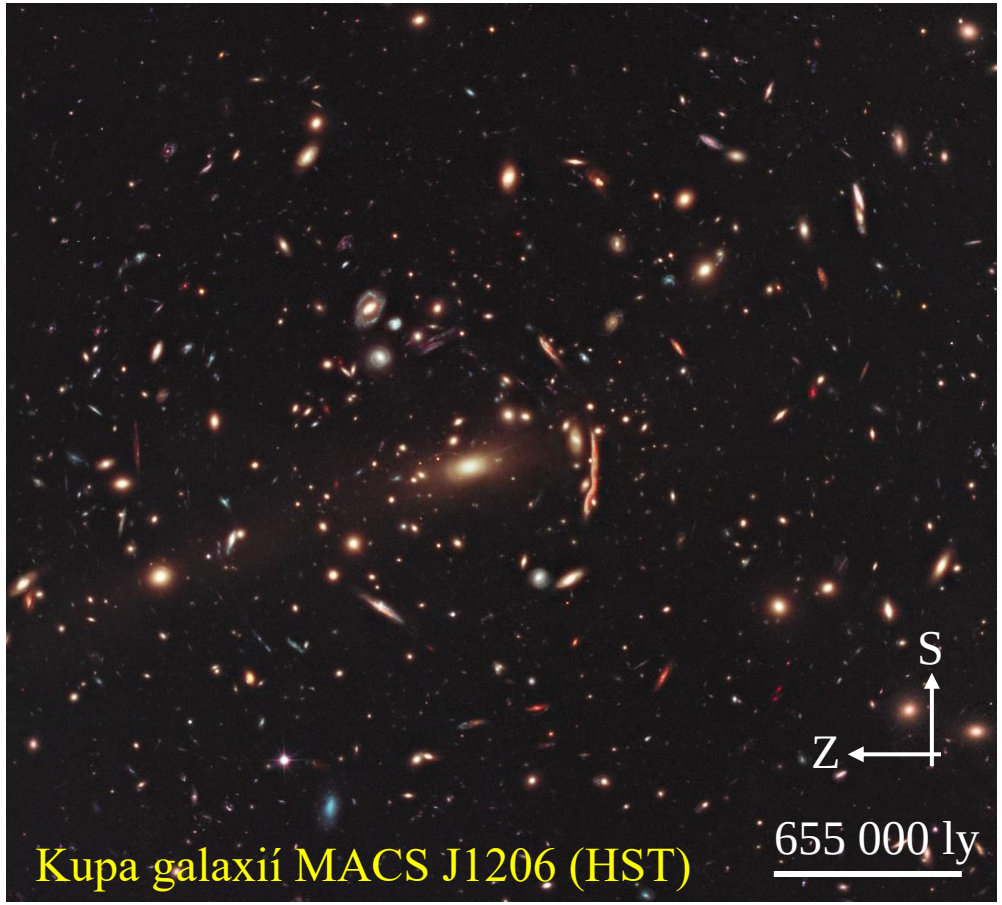
Numerické simulace

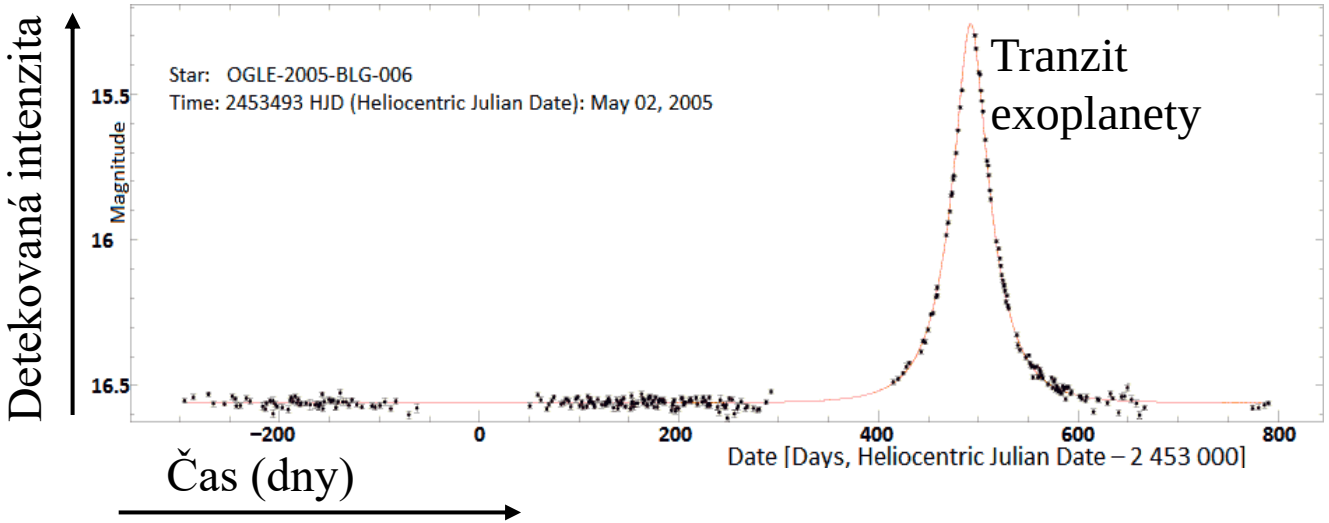
Gravitační čočka a mikročočkování



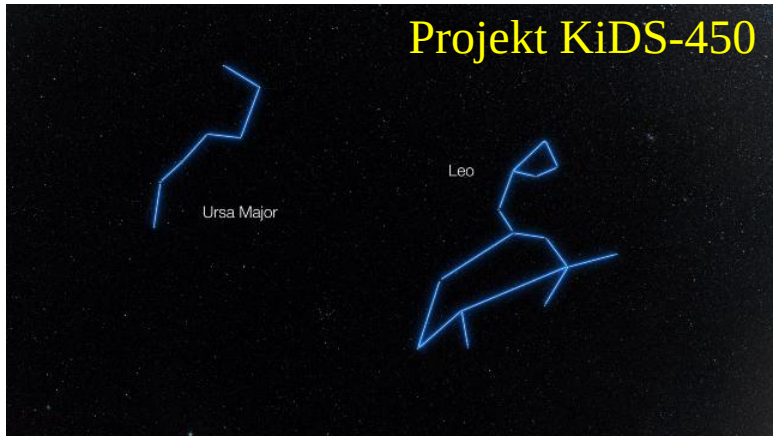


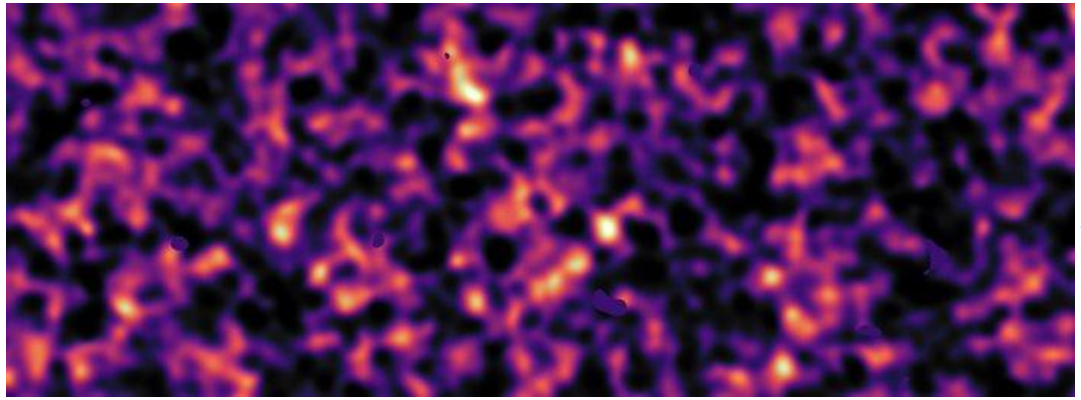


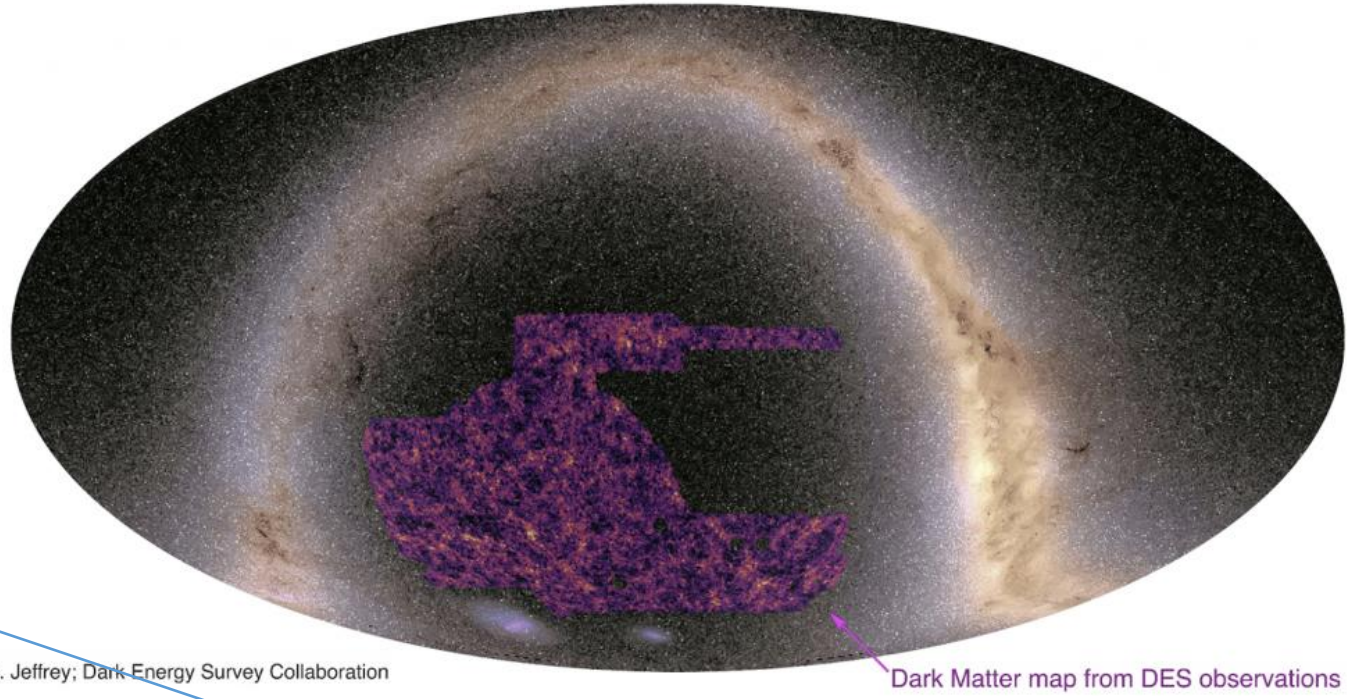


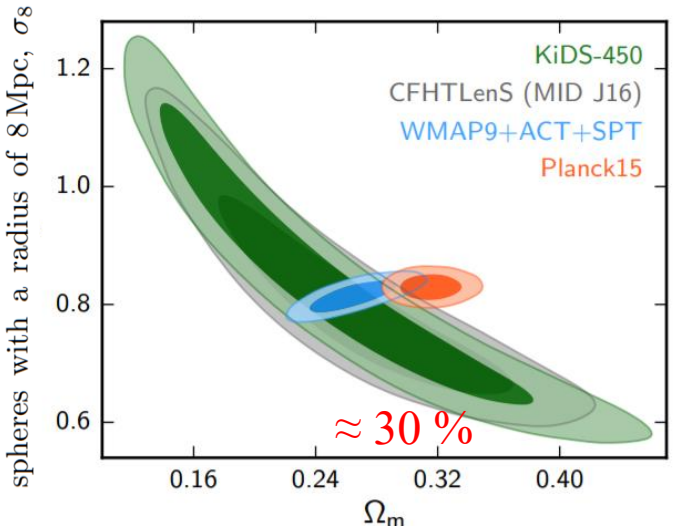


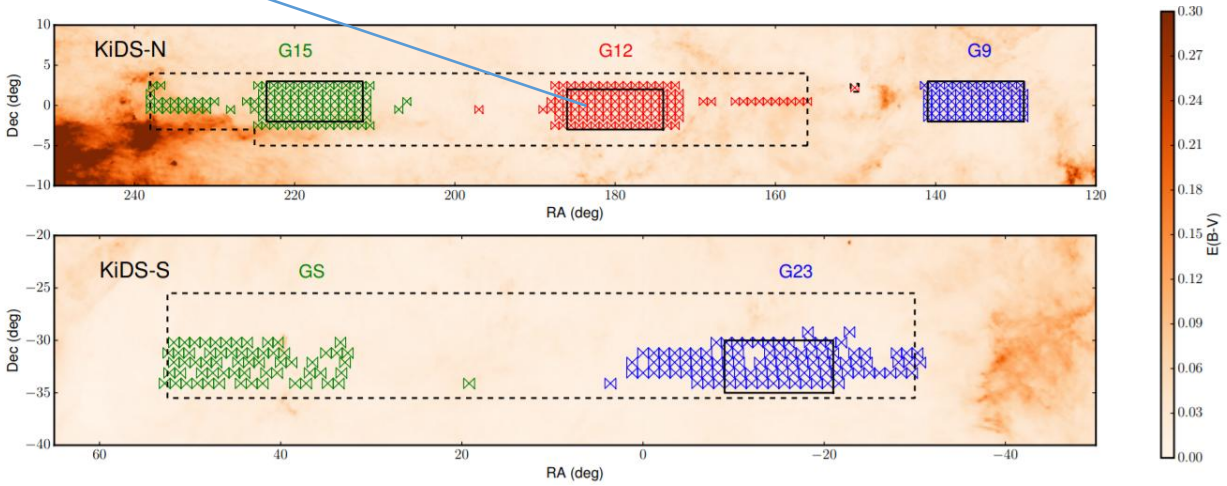
Mikročockování a rozložení Temné hmoty





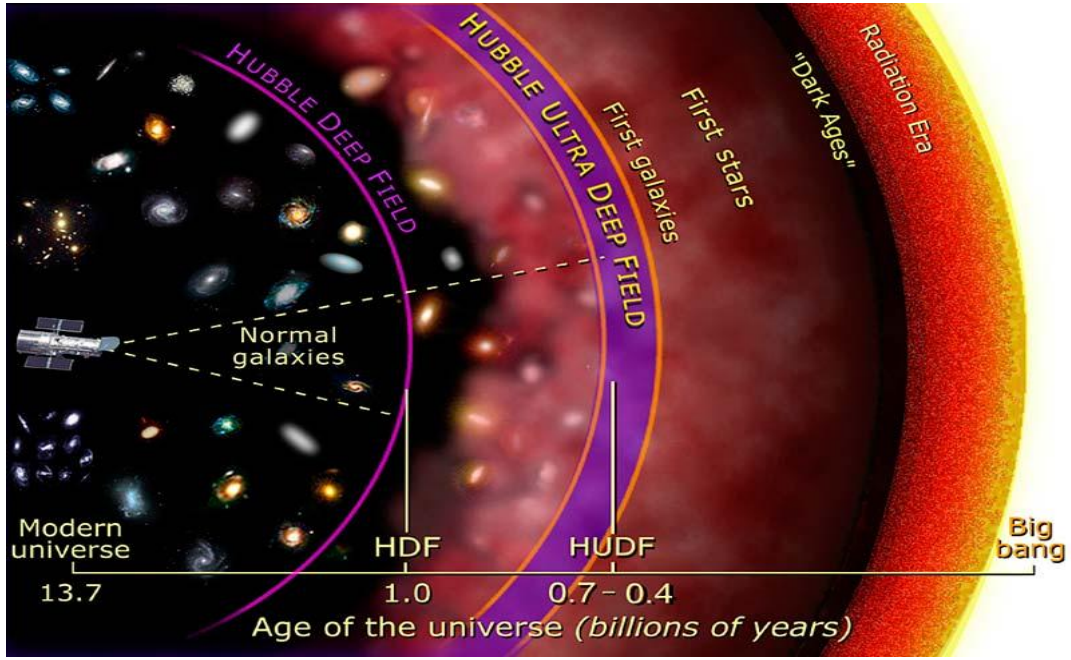


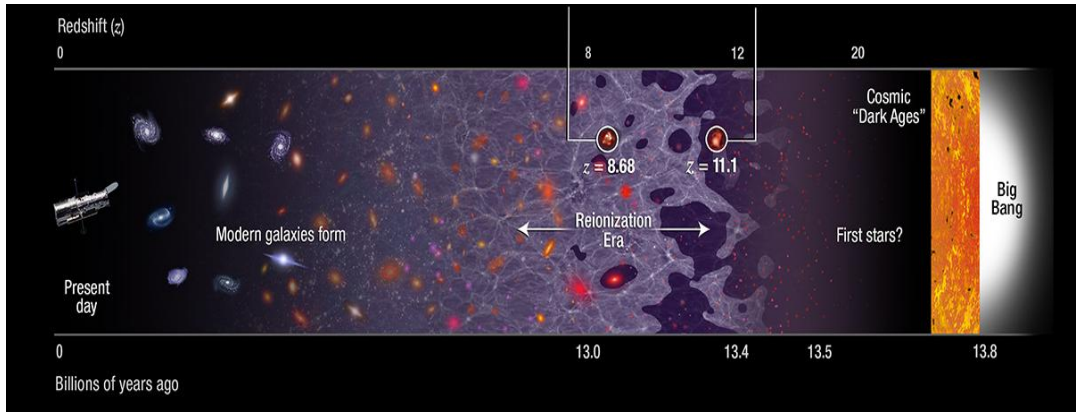


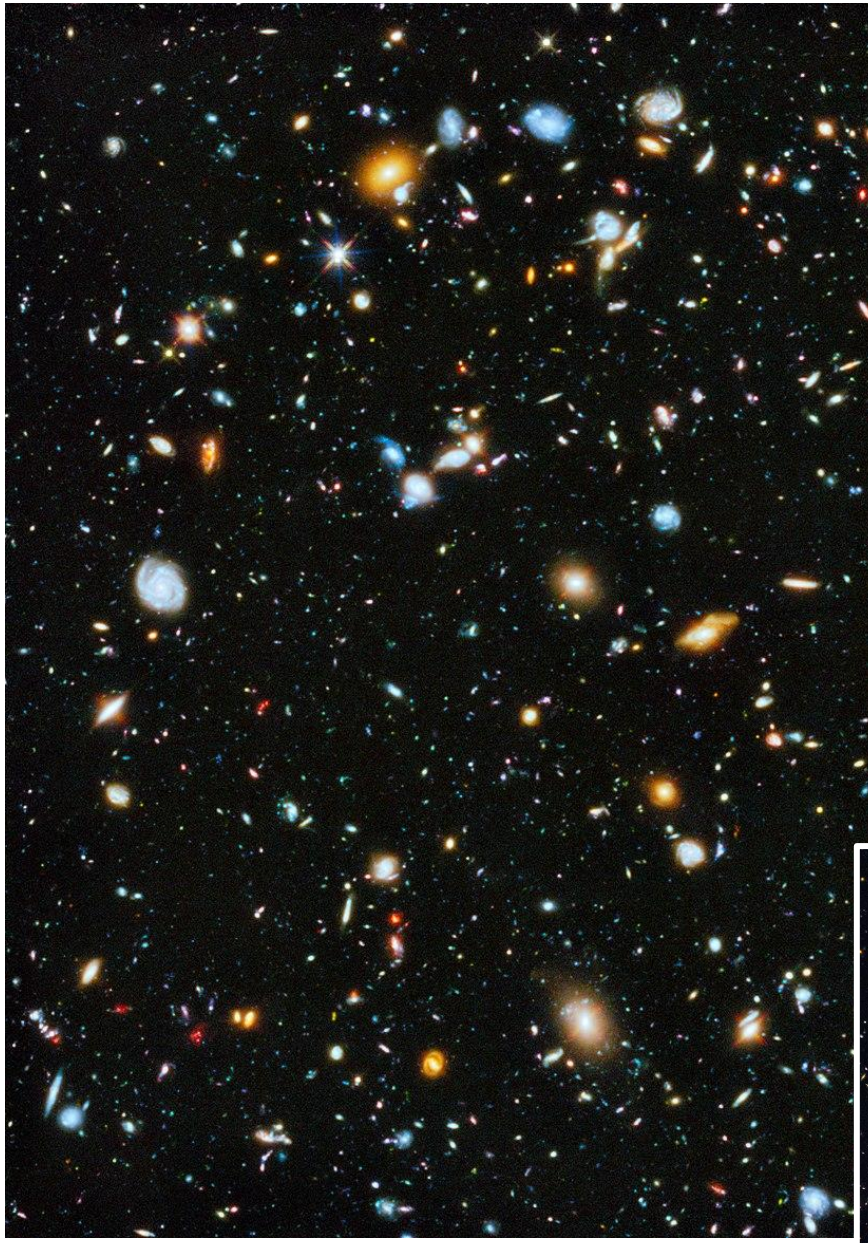


Pozorování dalekého Vesmíru

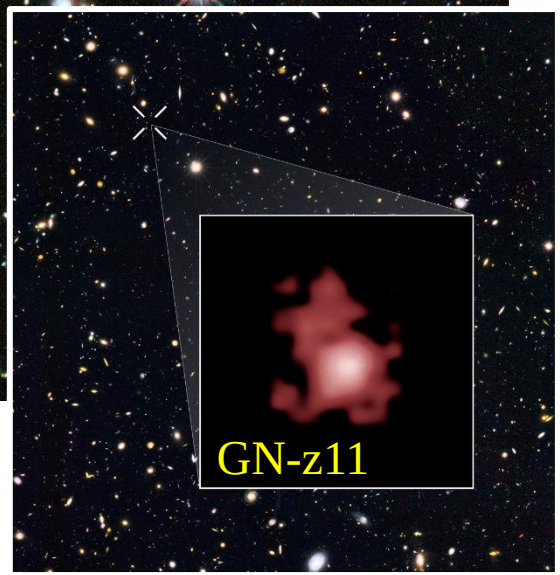
- Vznik galaxií 400 - 800 mil. let po Velkém třesku?
- Snímek ze souhvězdí Pece (jižní polokoule)
- Expozice téměř milion sekund
- Přes 10 000 galaxií

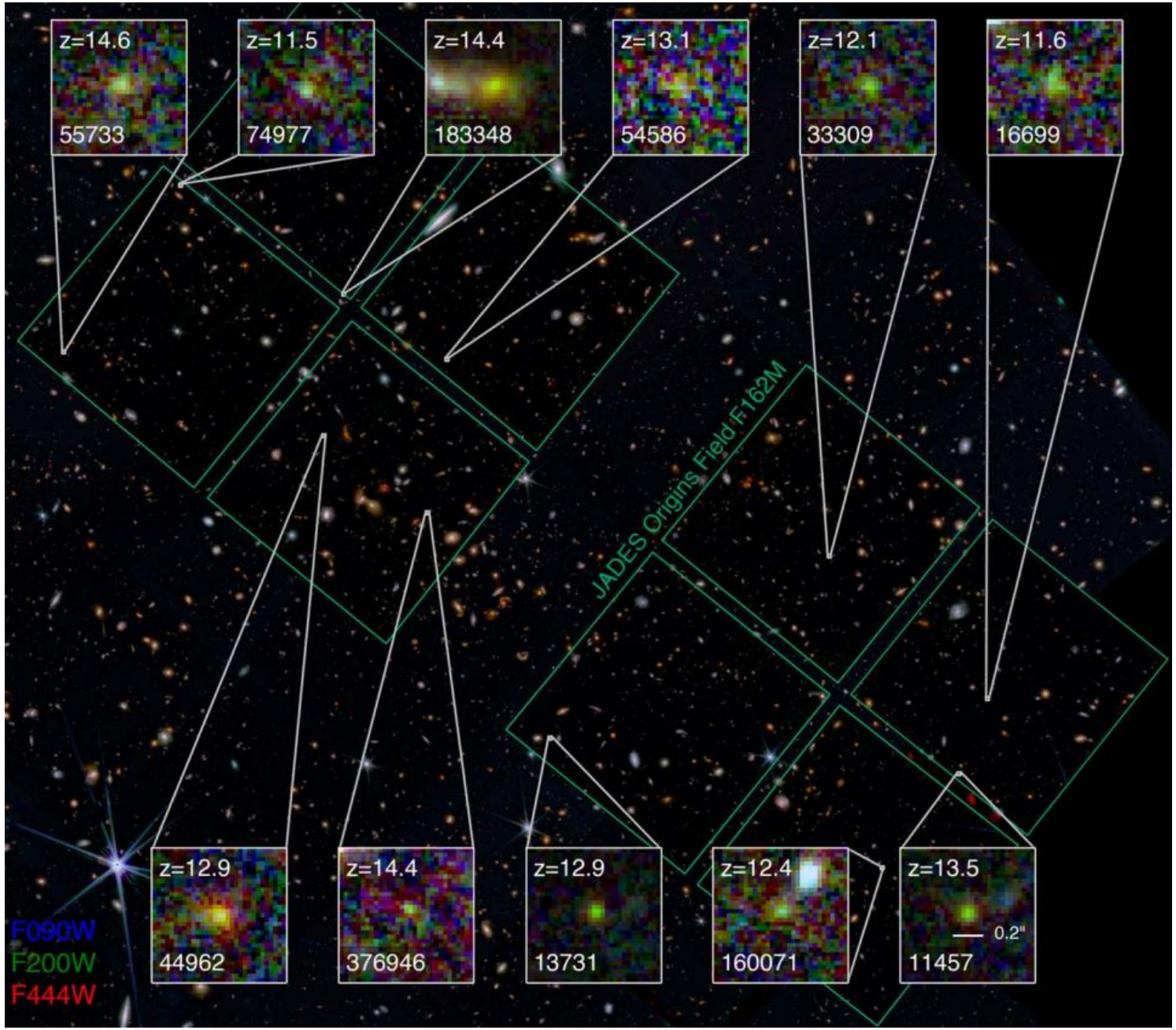


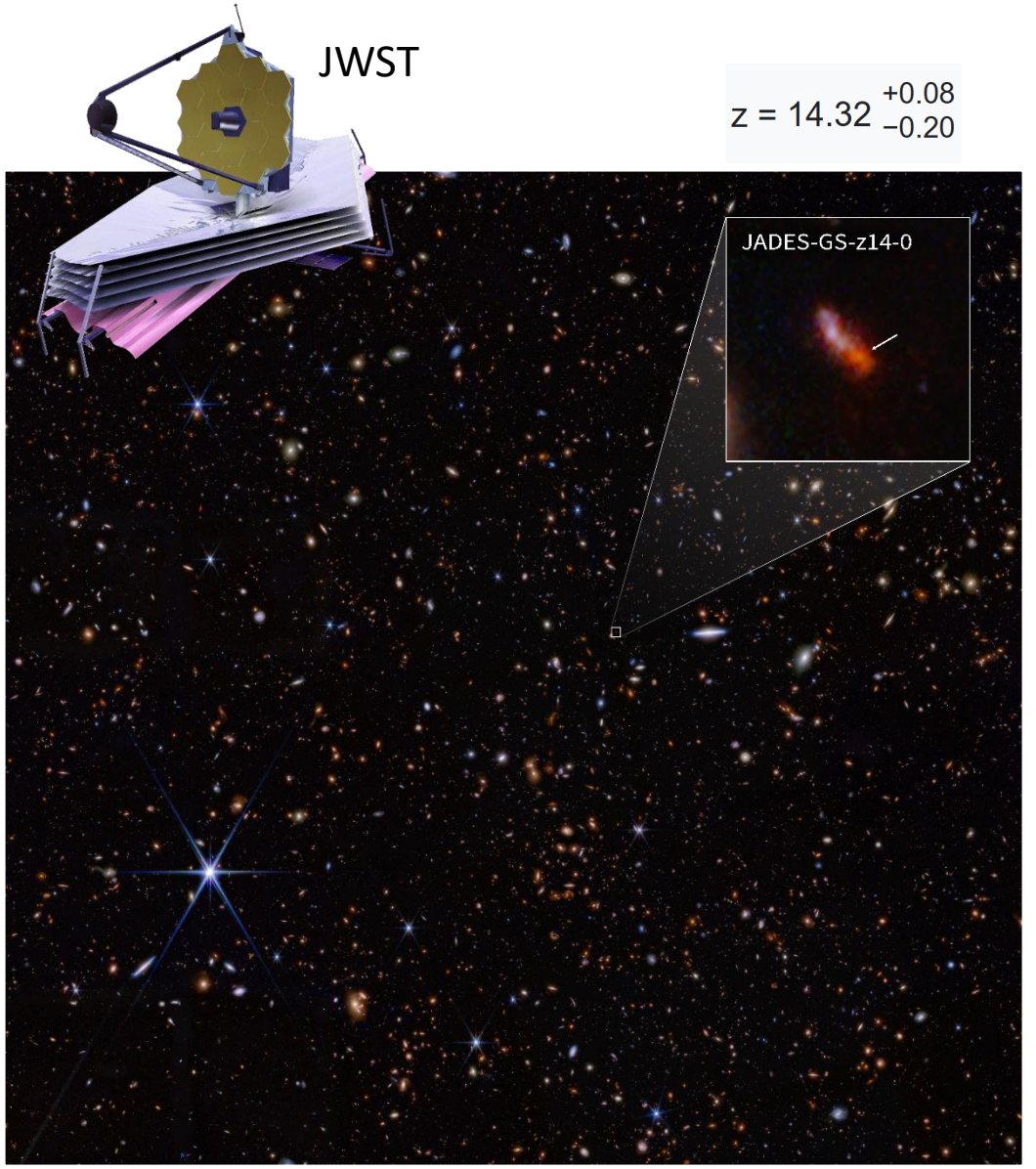




Hubbleovo ultrahluboké pole (2004)





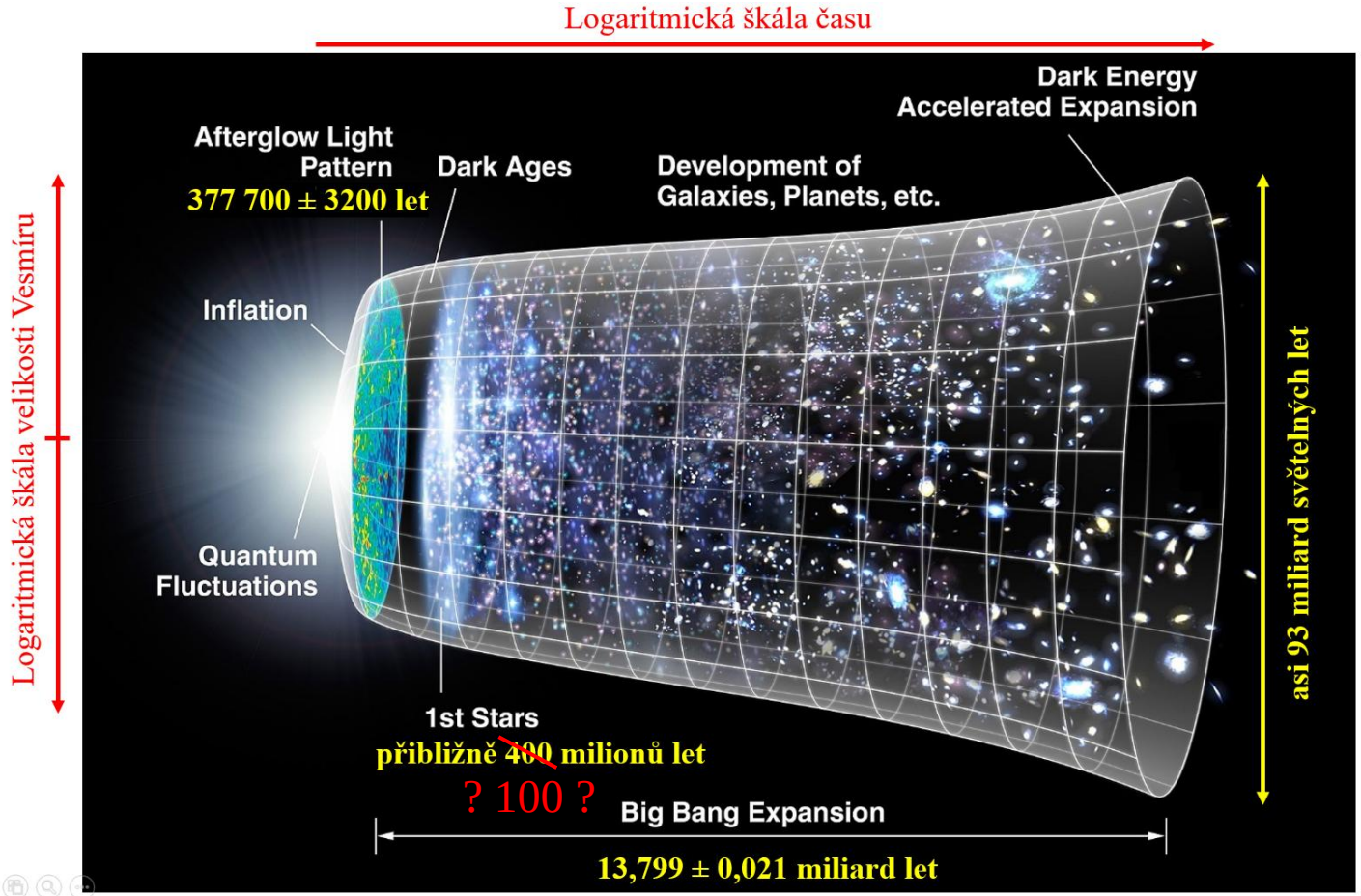


z = 14,32 odpovídá stáří Vesmíru **290 mil. let**

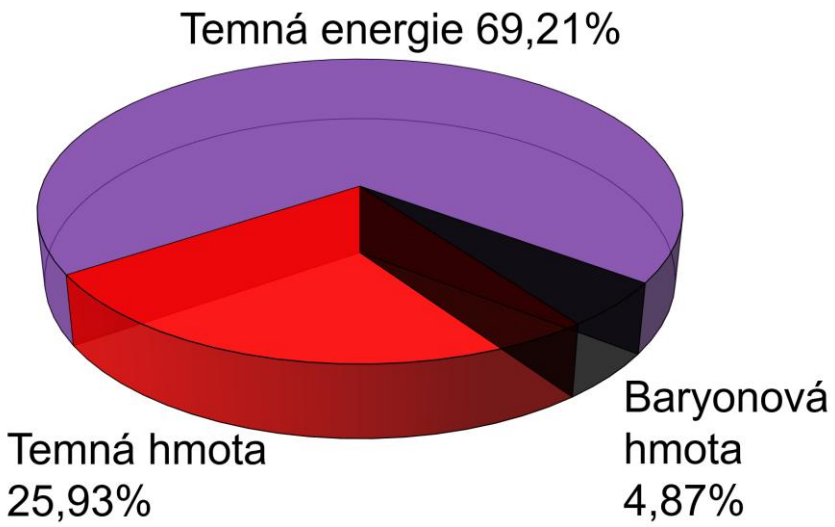
Vzdálenost ≈ **33,9 mld. ly**

Λ-CDM model Vesmíru = Standardní model Vesmíru

- Λ-*Cold Dark Matter* (zahrnuje v sobě Big Bang model, Temnou energii, Temnou hmotu)



(Dnes)

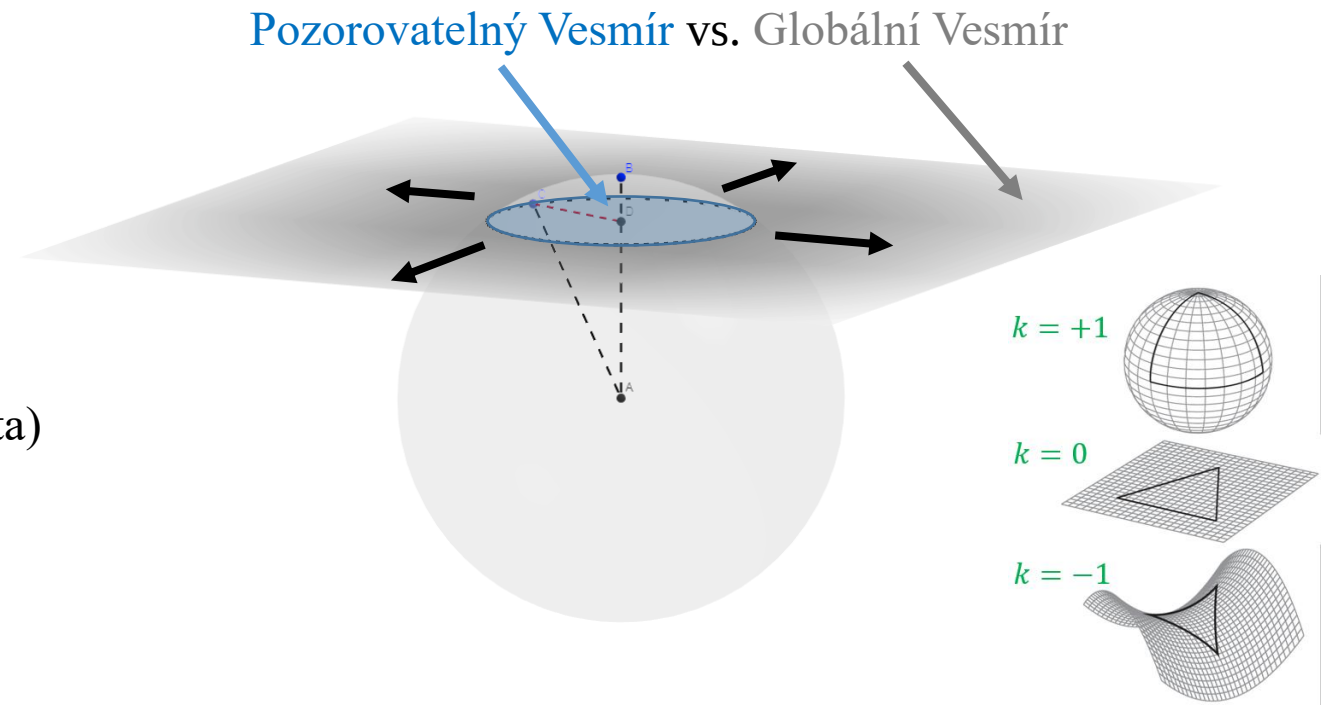


Experimentální důkazy:

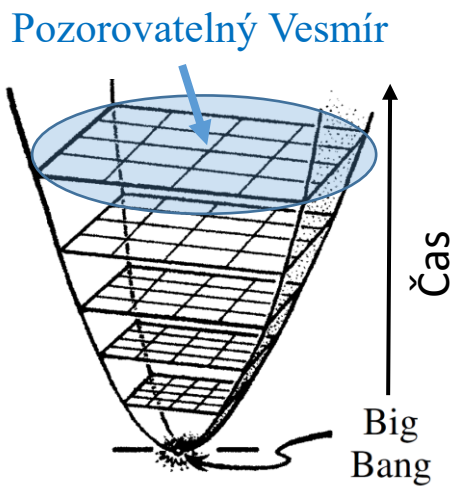
1) Zastoupení lehkých prvků ✓
2) Reliktní záření (CMB) ✓
3) Supernovy typu Ia (SNe) ✓
4) Baryonové akustické oscilace (BAO) ✓

A co teda ty hranice Vesmíru?

Stáří:	$13{,}8 \times 10^9$ let	
Poloměr:	$46{,}6 \times 10^9$ Ly	($4{,}4 \times 10^{26}$ m)
Objem:	$3{,}5 \times 10^{80}$ m^3	(koule)
Hmotnost:	$1{,}5 \times 10^{53}$ kg	(baryonová hmota)
Hustota:	$9{,}9 \times 10^{-27}$ kg/m^3	(6 protonů/m^3)
Teplota:	2,725 K	



- Globální Vesmír (prostor) se zvětšuje se zrychlením => „utíká“ Pozorovatelnému Vesmíru
- Horizont současných kosmických událostí, které se k nám ještě dostanou (R = 16 GLy)
- Konec pozorování ostatních galaxií (R = 62 GLy)
- Za 4 – 6 mld. let již neuvidíme galaxie s rudým posuvem z = 5
- Předpokládá se, že Globální Vesmír může být mnohonásobně větší – (prozatím) NEVÍME!

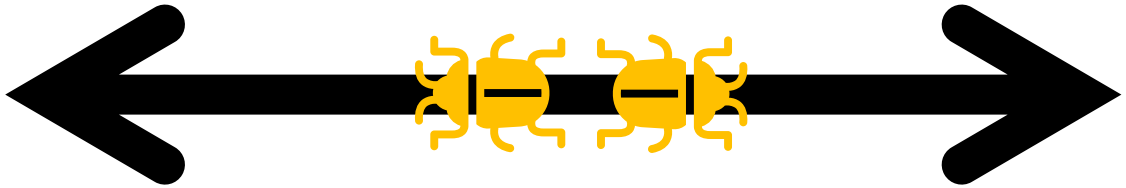


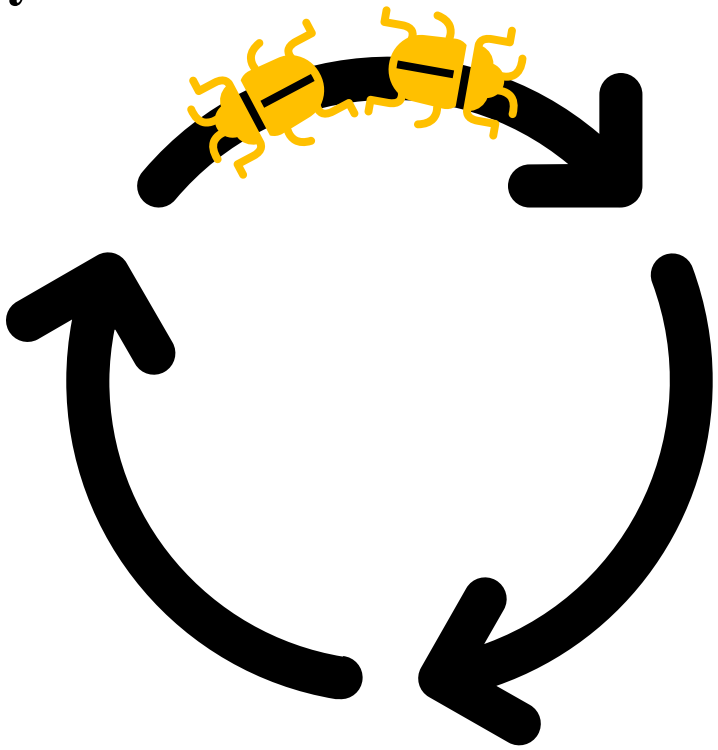
Vesmír je homogenní, izotropní, exponenciálně zrychlující, (velmi) plochý, (zřejmě) konečný bez pevné hranice

Topologie Globálního vesmíru?

Konečný Vesmír bez hranice? aneb Dimenze Vesmíru

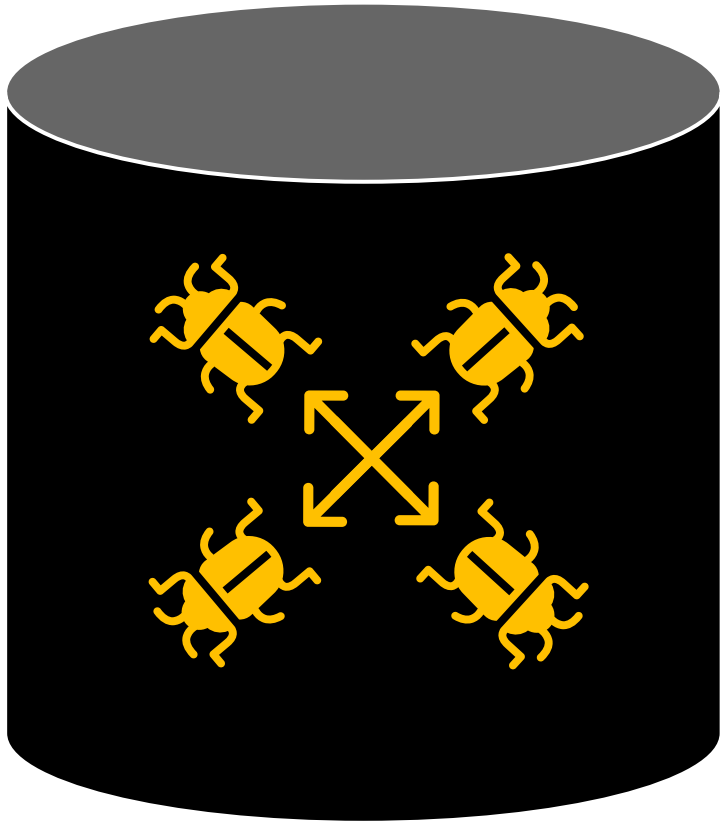
1D Vesmíry

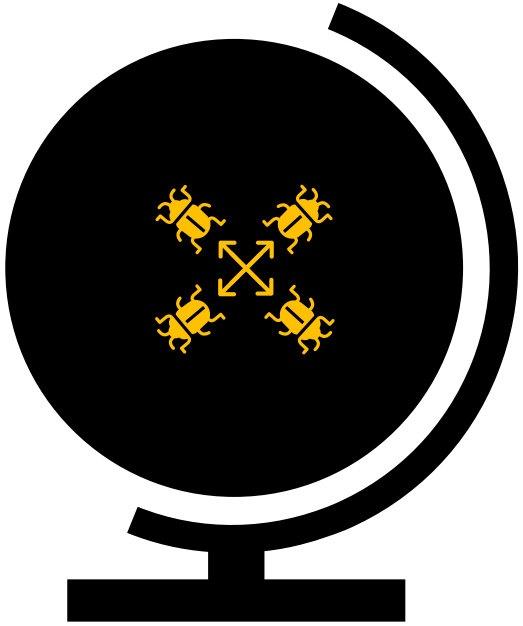




Konečný Vesmír bez hranice? aneb Dimenze Vesmíru

2D Vesmíry

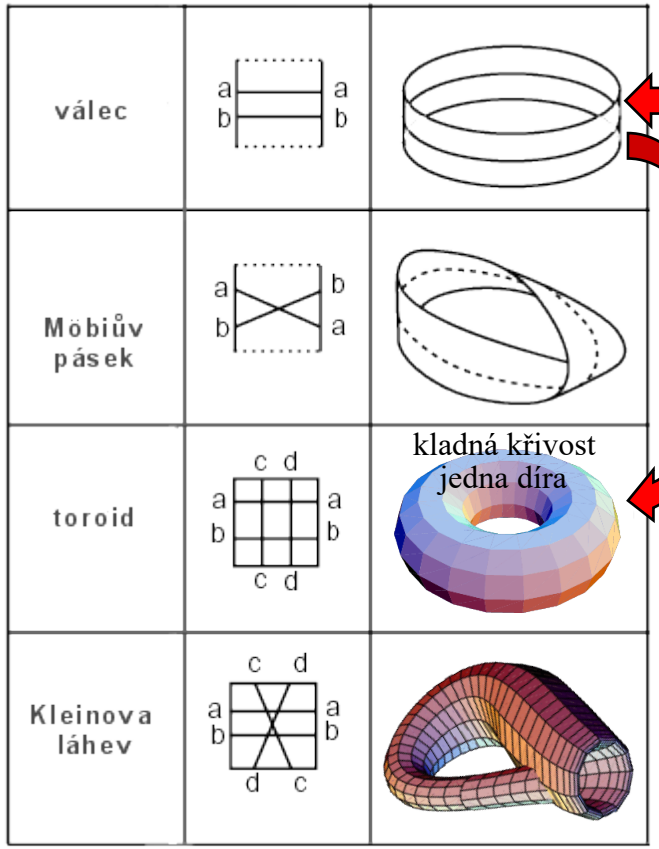


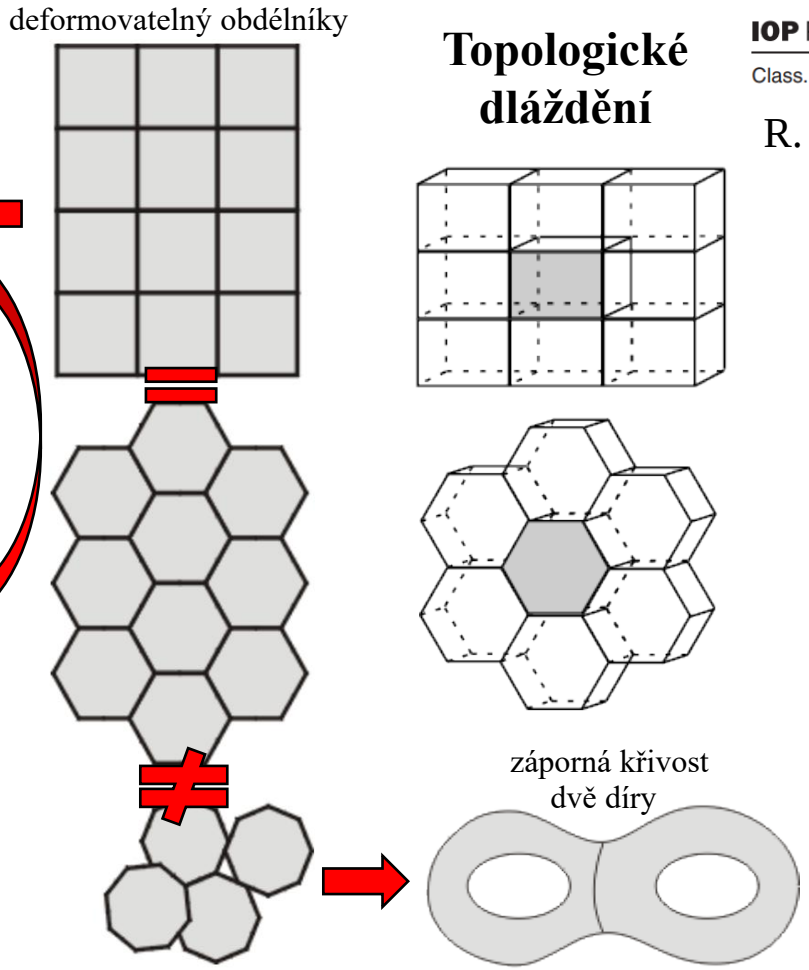


Konečný Vesmír bez hranice? aneb Dimenze Vesmíru

3D Vesmír(y)?

Topologie Vesmíru





Topologické dláždění

IOP Publishing Classical and Quantum Gravity
Class. Quantum Grav. **38** (2021) 225005 (31pp) https://doi.org/10.1088/1361-6382/ac27f0

R. Aurich, T. Buchert, M.J. France, F. Steiner

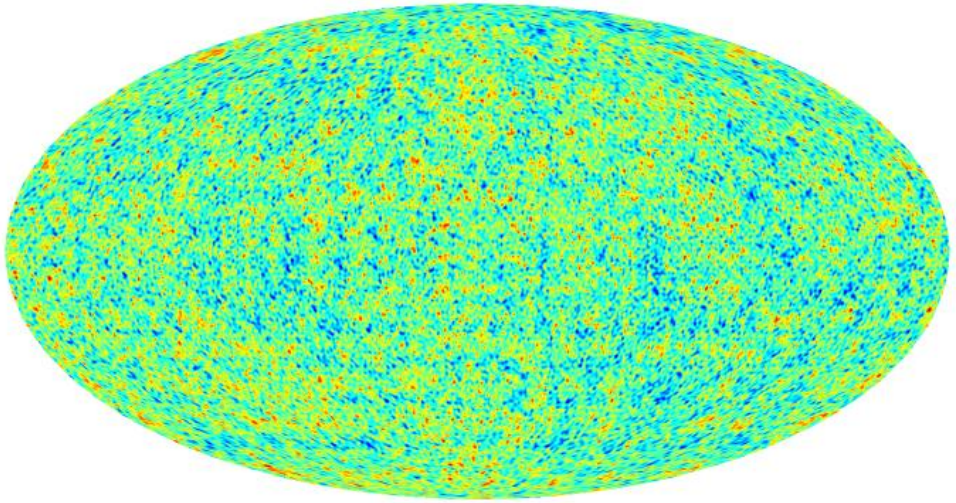


Figure 6: This figure shows a CMB sky map for a small cubic 3−torus fundamental cell of $L = 0.5L_H$. The resolution parameters are $N_{\rm side} = 256$, $l_{\rm range} = [2, 256]$ and $\vartheta_G = (1/3)^\circ$.

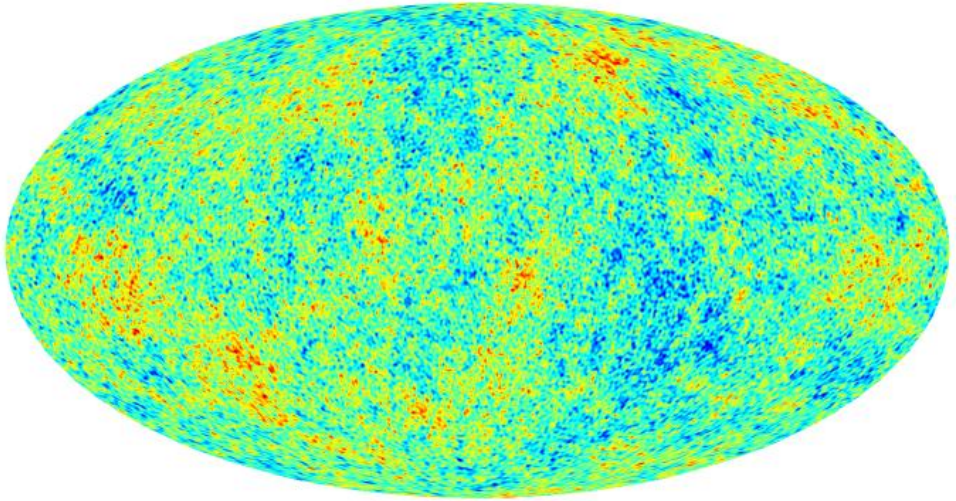
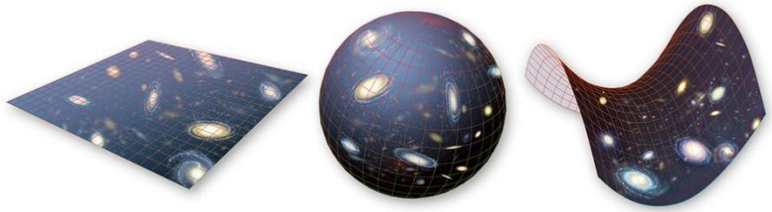
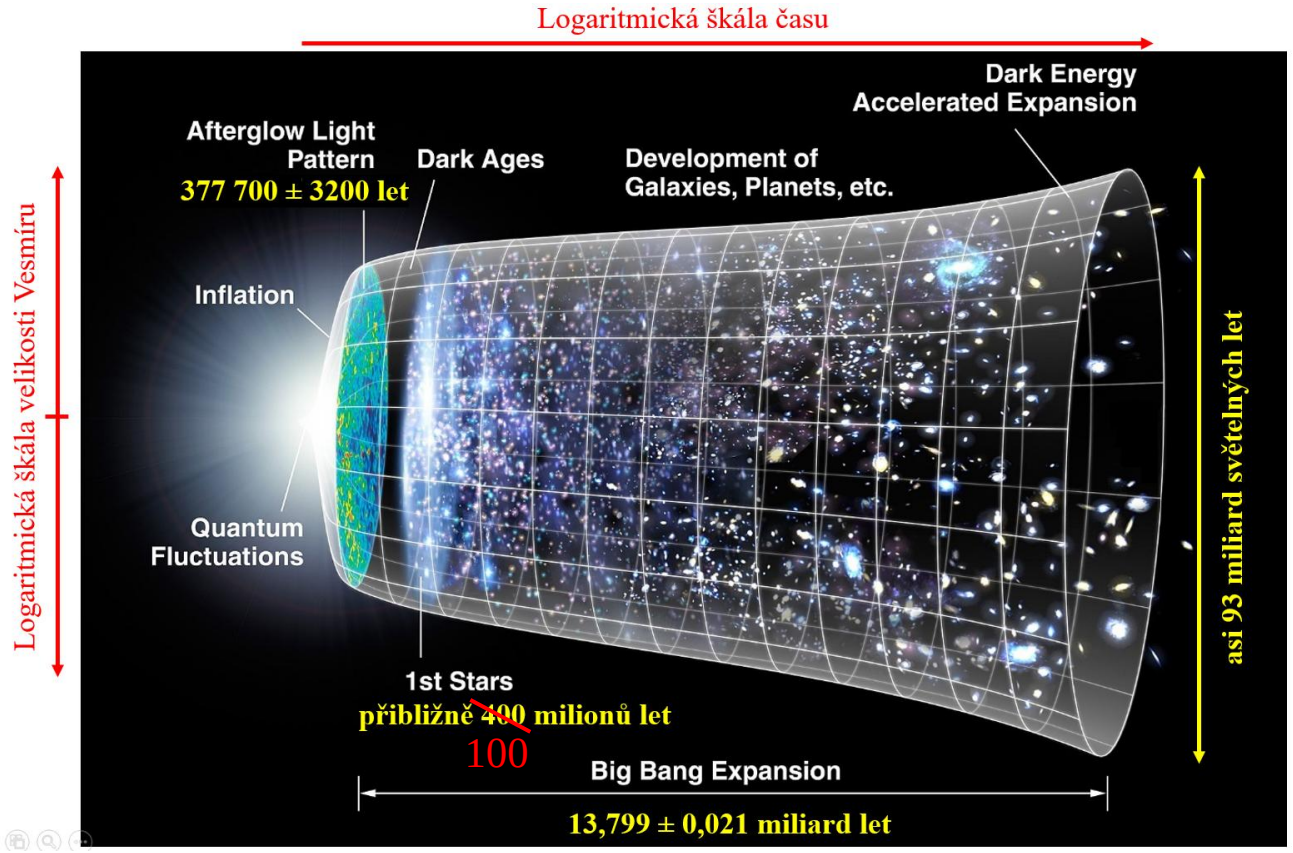


Figure 7: Figure showing a CMB sky map for a cubic 3−torus fundamental cell of $L = 3.0L_H$, which is six times larger than the one used for figure 6. The resolution parameters are those of the figure 6.

(velmi) plochý
(zřejmě) konečný bez pevné hranice



Závěr

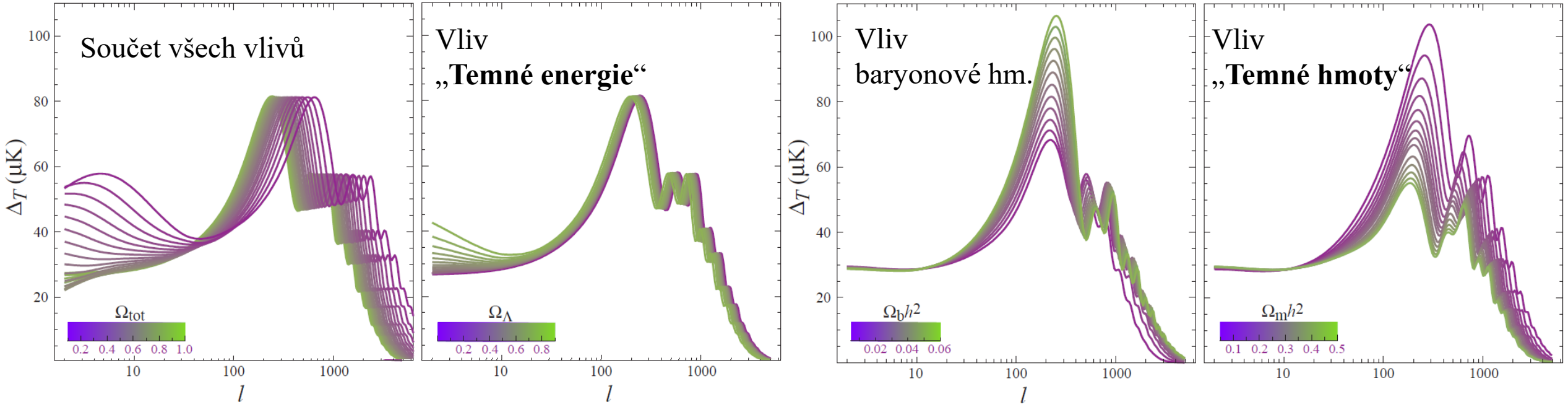


Děkuji za pozornost!

petr.viewegh@vut.cz

Λ-CDM model Vesmíru

Hu W., Dodelson S. *Cosmic microwave background anisotropies*, (2002)

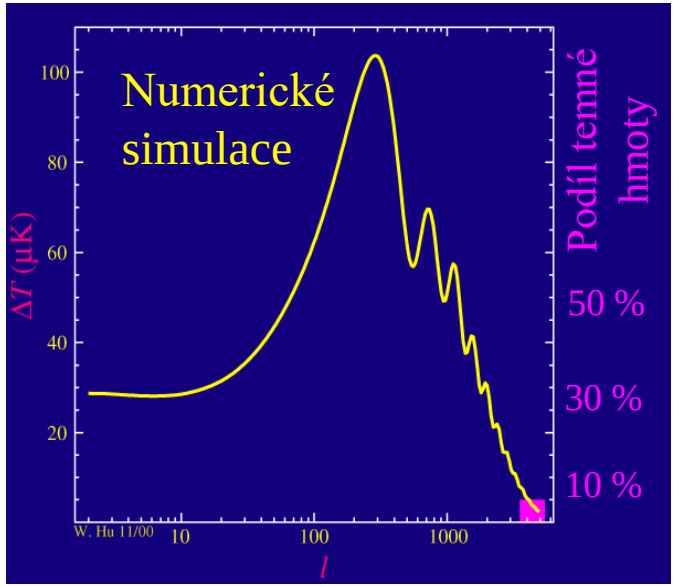


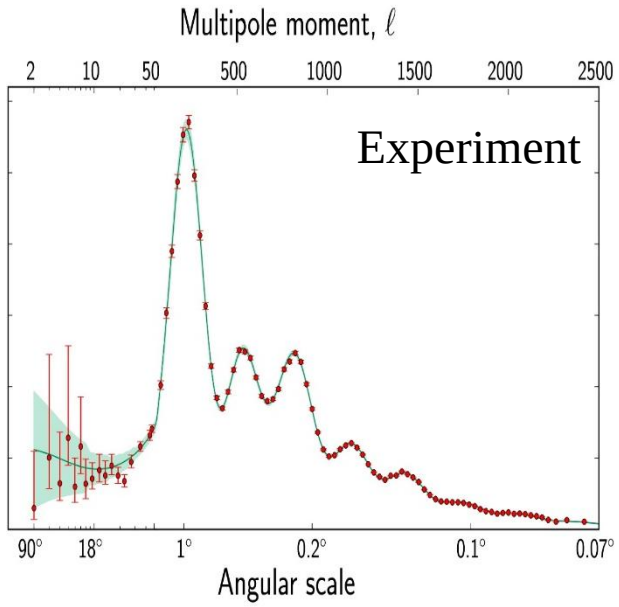
Baryonová hmota + neznáma hmota

$$\Omega_{total} = \Omega_\Lambda + \Omega_{mass} + \Omega_{relativistic}$$

$$\Omega_{total} = \frac{\rho}{\rho_{crit}} \qquad \rho_{crit}(t) = \frac{3H^2(t)}{8\pi G}$$

$$H(t) = \frac{\dot{R}(t)}{R(t)} = H_0 \sqrt{\frac{\Omega_{mass}}{R^3(t)} + \frac{\Omega_{rel}}{R^4(t)} + \Omega_\Lambda}$$



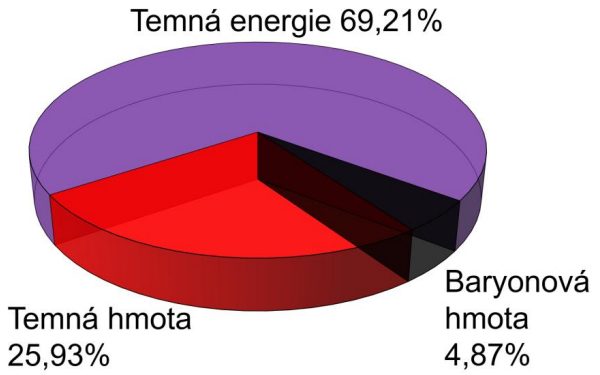


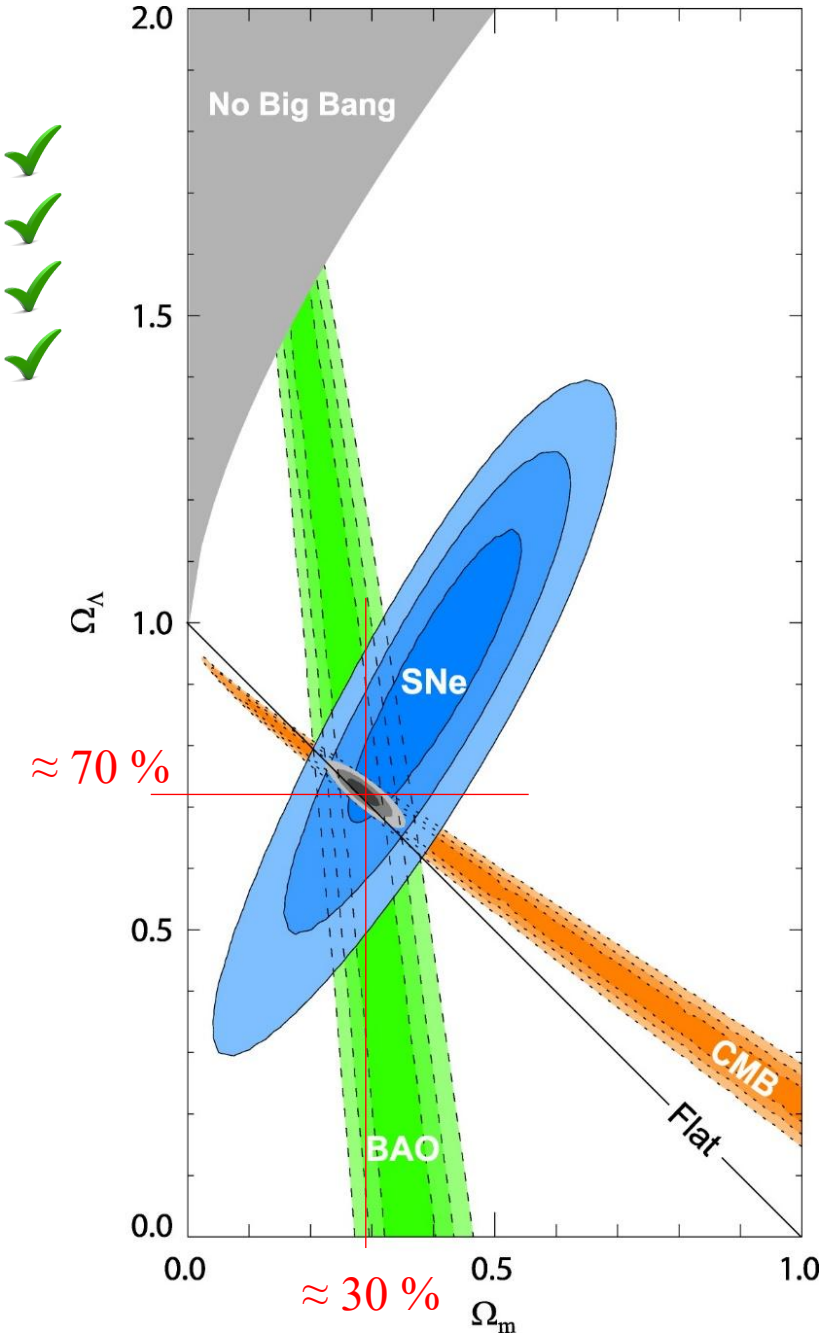
http://background.uchicago.edu/~whu/intermediate/intermediate.html

Λ-CDM model Vesmíru

1) Zastoupení lehkých prvků ✓
2) Reliktní záření (CMB) ✓
3) Supernovy typu Ia (SNe) ✓
4) Baryonové akustické oscilace (BAO) ✓

(Dnes)

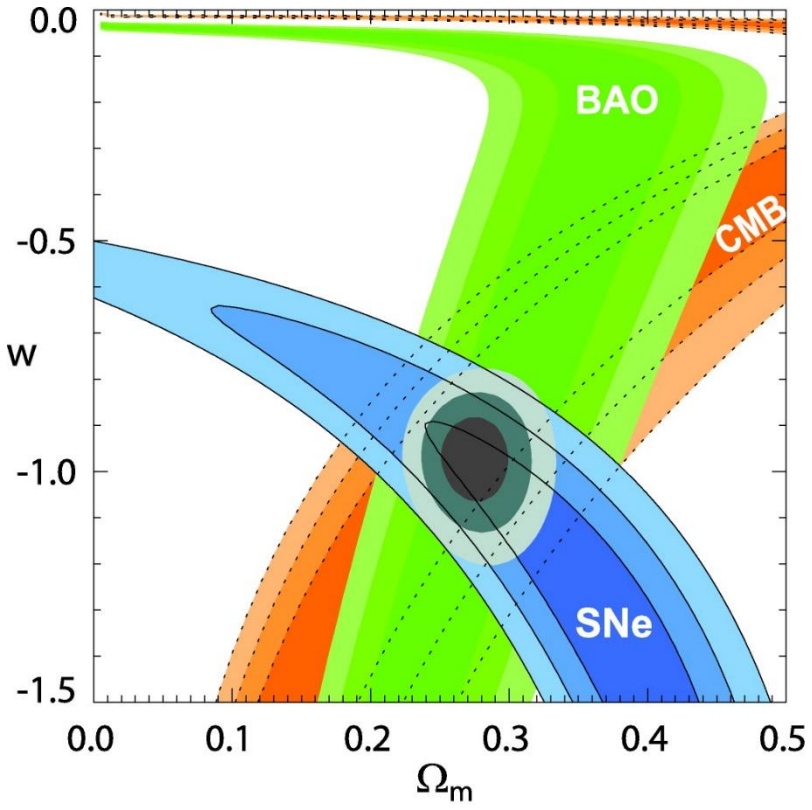




Stavová rovnice Temné energie: $p = w\rho$

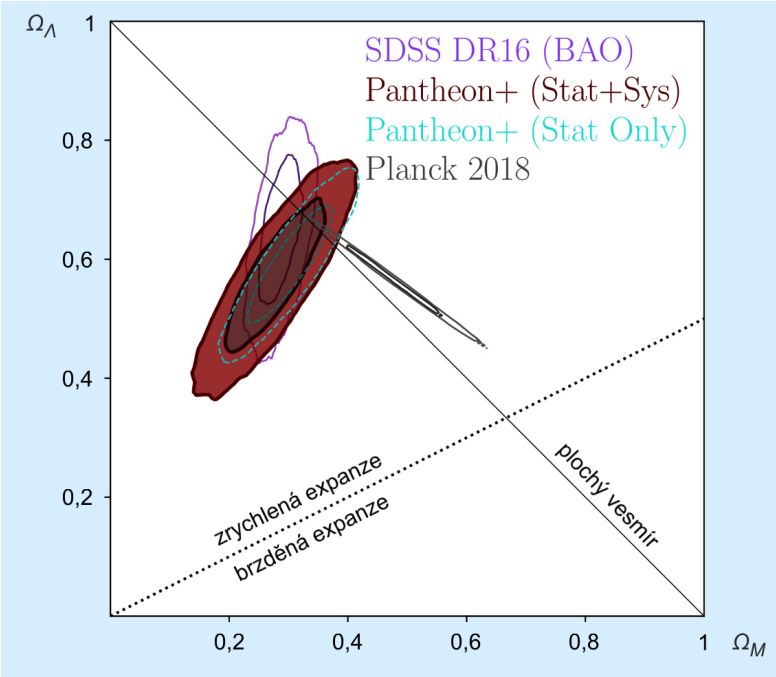
pro $w = -1$

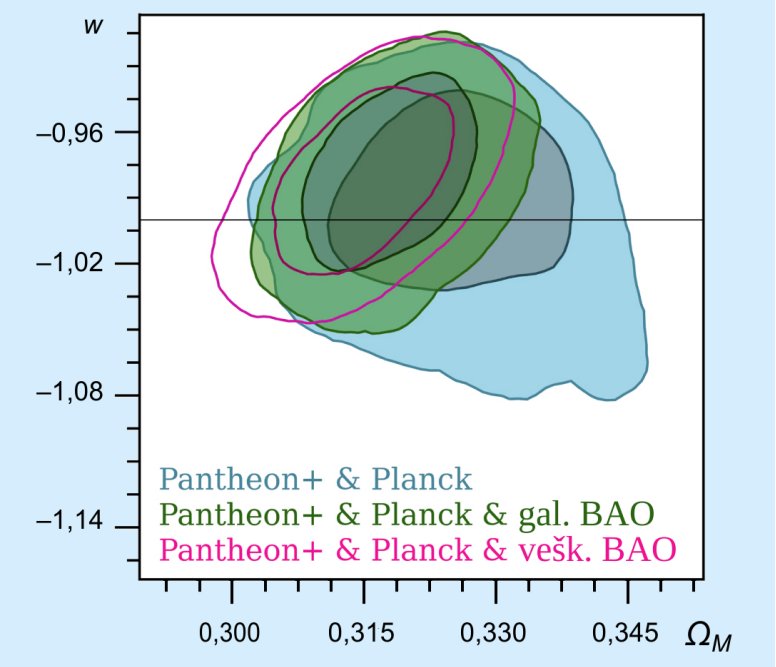
Temná energie <=> kosmologická konstanta Λ



R. Amanullah *et al.*, ApJ, **716,** (2010)

Λ-CDM model Vesmíru

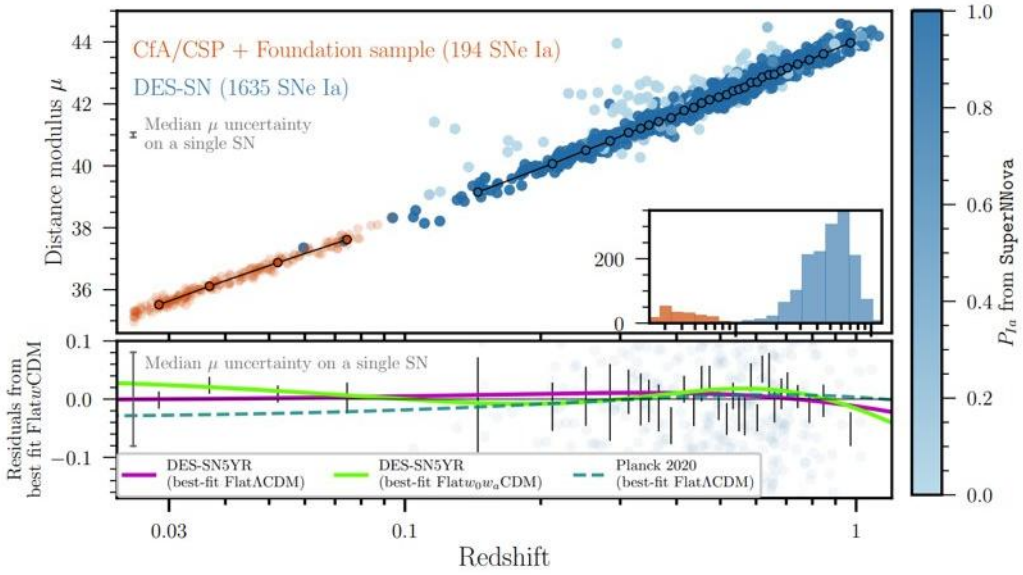


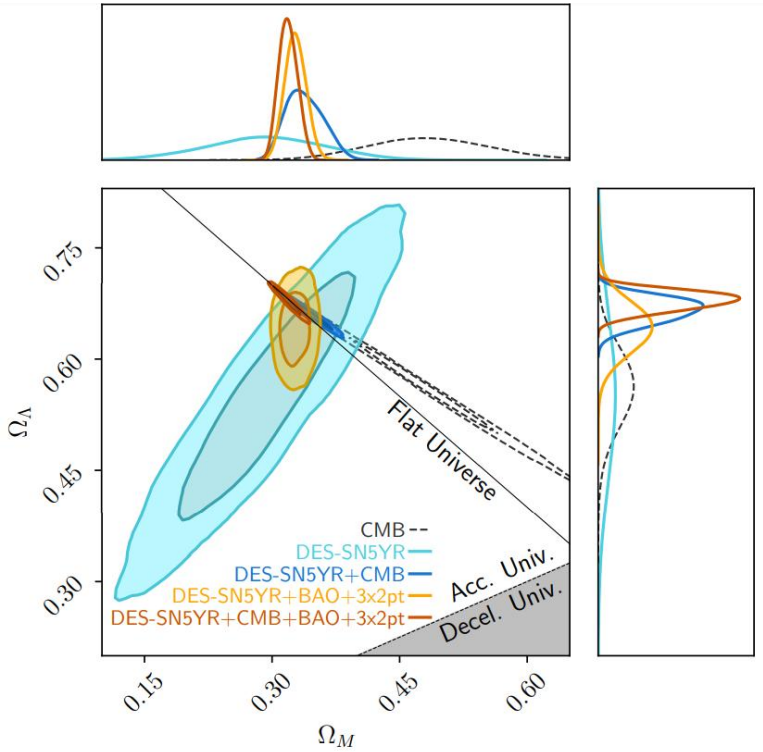


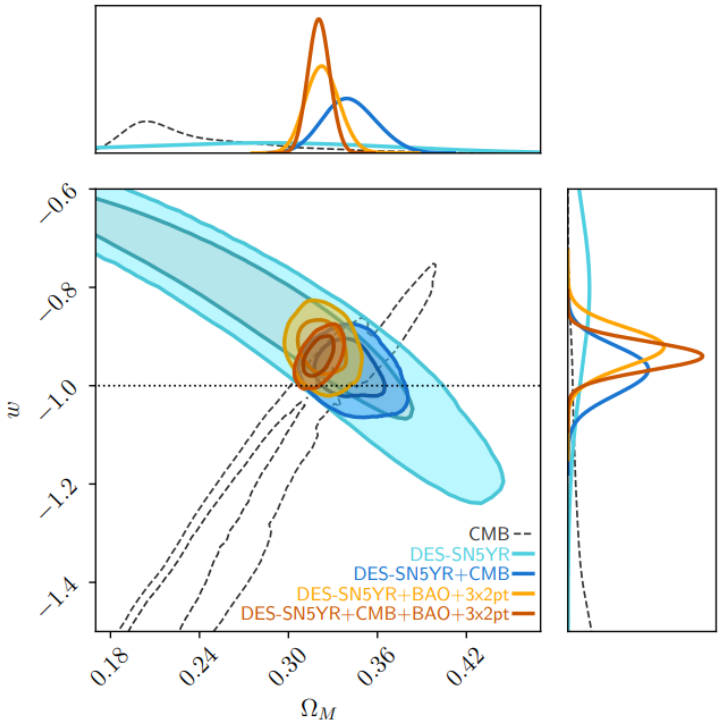
Projekt: **Pantheon plus** (2022)

- více jak 1550 SNIa, z = 0,001 až 2,26

Projekt: **Dark Energy Survey** (2024)



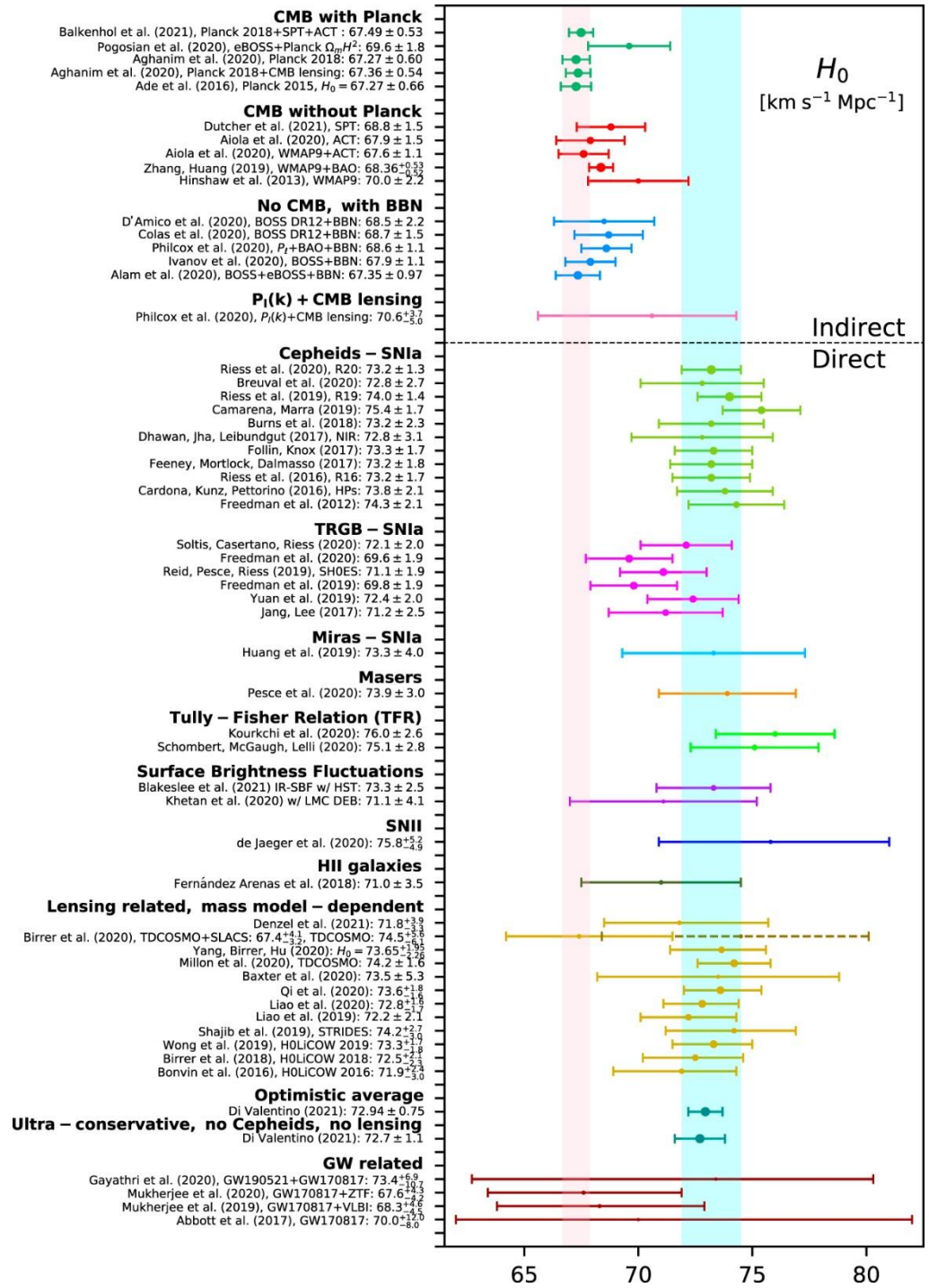


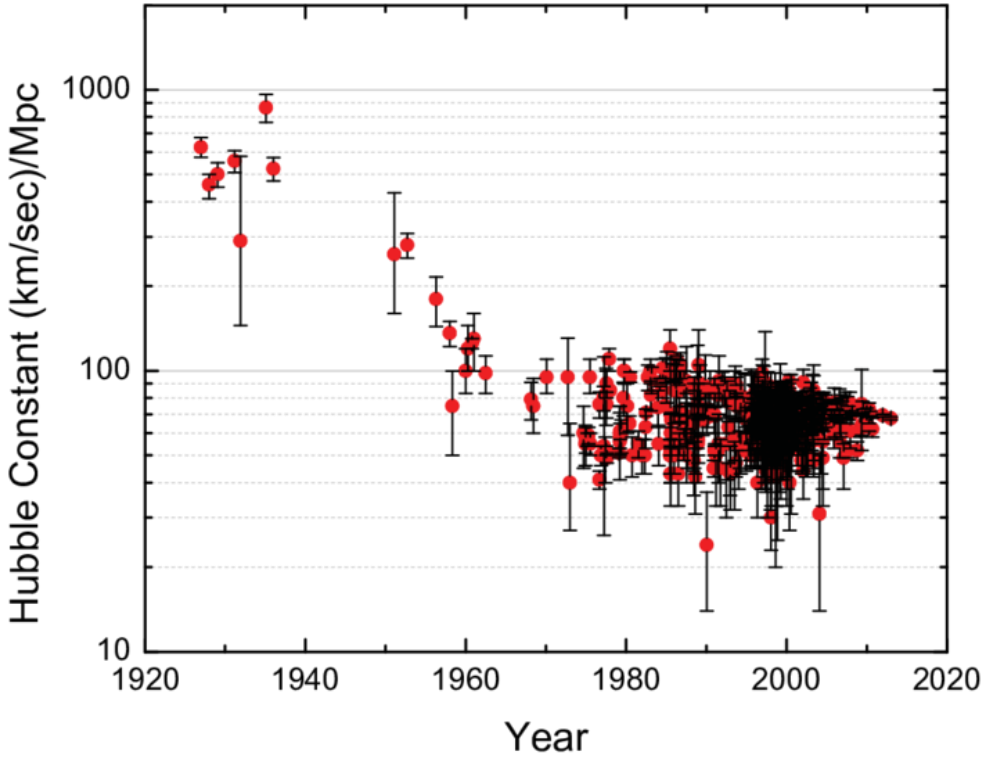


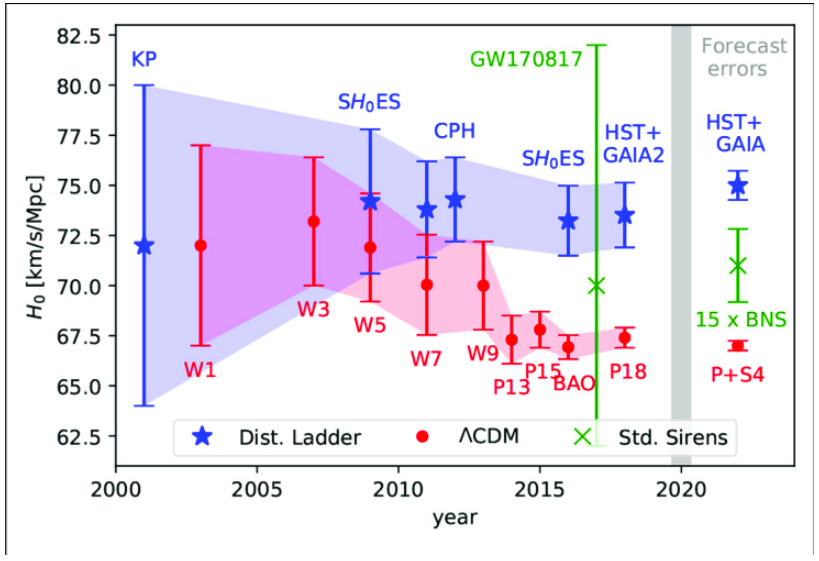
Brout *et al.*, ApJ, **938,** (2022)

https://arxiv.org/pdf/2401.02929.pdf

„Hubbleovo napětí“







CMB: **67,4 ± 0,5** km s^{-1}Mpc^{-1}

SNIa: **73,5 ± 1,1** km s^{-1}Mpc^{-1}